

Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/84-4.2.1.1-a+b-cos-ⁿ

Nasser M. Abbasi

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [62]. This is test number [84].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (62)	0.00 (0)
Mathematica	100.00 (62)	0.00 (0)
Fricas	72.58 (45)	27.42 (17)
Maple	72.58 (45)	27.42 (17)
Giac	62.90 (39)	37.10 (23)
Maxima	62.90 (39)	37.10 (23)
Mupad	56.45 (35)	43.55 (27)
Sympy	51.61 (32)	48.39 (30)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

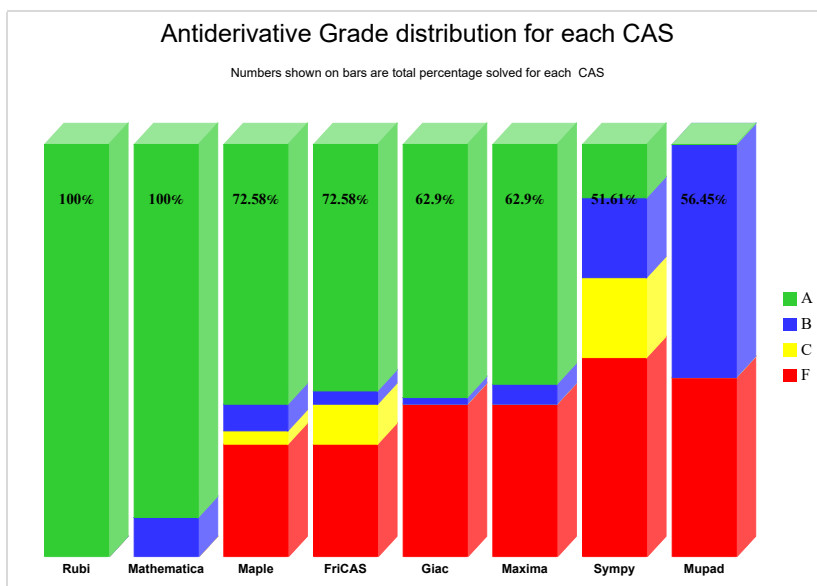
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

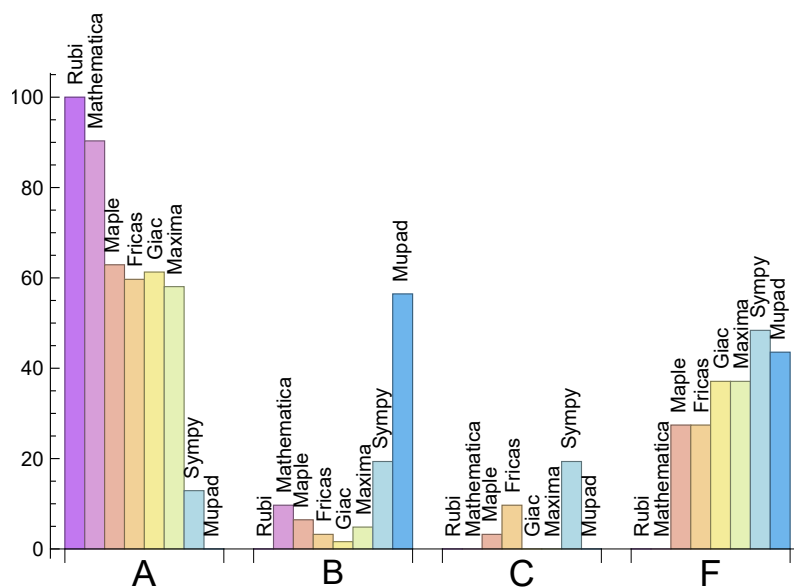
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	90.32	9.68	0.00	0.00
Maple	62.90	6.45	3.23	27.42
Giac	61.29	1.61	0.00	37.10
Fricas	59.68	3.23	9.68	27.42
Maxima	58.06	4.84	0.00	37.10
Sympy	12.90	19.35	19.35	48.39
Mupad	N/A	56.45	0.00	43.55

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	17	100.00 %	0.00 %	0.00 %
Fricas	17	100.00 %	0.00 %	0.00 %
Giac	23	100.00 %	0.00 %	0.00 %
Maxima	23	100.00 %	0.00 %	0.00 %
Sympy	30	96.67 %	0.00 %	3.33 %
Mupad	27	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

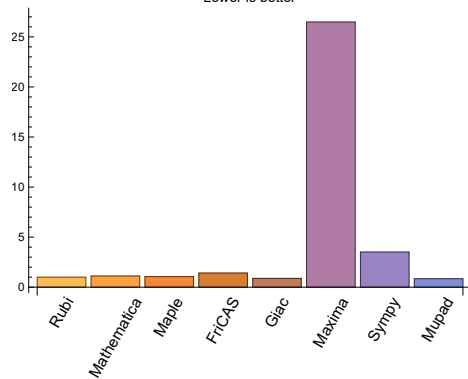
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	88.03	1.00	83.00	1.00
Mathematica	0.13	104.81	1.12	69.00	1.00
Maple	0.08	107.67	1.06	72.00	0.82
Maxima	1.04	2655.69	26.49	94.00	1.19
Fricas	0.34	135.69	1.41	90.00	1.11
Sympy	1.36	346.69	3.51	299.50	3.76
Giac	0.46	70.13	0.88	75.00	0.83
Mupad	0.38	65.26	0.85	67.00	0.90

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.

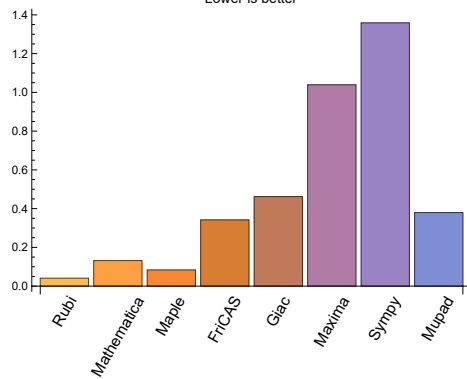
Normalized mean size of antiderivative

Lower is better



Mean time used (seconds)

Lower is better



1.4 list of integrals that has no closed form antiderivative

{

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

The following fields are present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 46, 47, 48, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62 }

B grade: { 37, 41, 45, 49, 56, 61 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 7, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 55 }

B grade: { 6, 50, 51, 52 }

C grade: { 5, 53 }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 56, 57, 58, 59, 60, 61, 62 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade: { 5, 6, 7 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade: { 6, 7 }

C grade: { 50, 51, 52, 53, 54, 55 }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 56, 57, 58, 59, 60, 61, 62 }

2.1.6 Sympy

A grade: { 18, 22, 26, 30, 34, 38, 42, 46 }

B grade: { 35, 36, 37, 39, 40, 41, 43, 44, 45, 47, 48, 49 }

C grade: { 19, 20, 21, 23, 24, 25, 27, 28, 29, 31, 32, 33 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 7, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade: { 5 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbrevi-

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA .	grade	A	A	A	A	A	A	F(-2)	A	F
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	119	119	83	86	77	75	0	108	-1
	N.S.	1	1.00	0.70	0.72	0.65	0.63	0.00	0.91	-0.01
	time (sec)	N/A	0.051	0.175	0.079	0.534	0.388	0.000	0.598	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	60	62	0	84	-1
N.S.	1	1.00	0.80	0.82	0.67	0.70	0.00	0.94	-0.01
time (sec)	N/A	0.033	0.085	0.089	0.564	0.351	0.000	0.470	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	38	44	0	55	-1
N.S.	1	1.00	0.93	0.98	0.64	0.75	0.00	0.93	-0.02
time (sec)	N/A	0.020	0.050	0.075	0.541	0.361	0.000	0.467	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.009	0.020	0.000	0.529	0.376	0.000	0.491	0.002

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	54	90	126	0	93	45
N.S.	1	1.00	0.87	1.17	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.015	0.012	0.027	0.545	0.379	0.000	0.491	0.002

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	138	15721	153	0	112	-1
N.S.	1	1.00	0.82	1.79	204.17	1.99	0.00	1.45	-0.01
time (sec)	N/A	0.030	0.060	0.088	1.159	0.366	0.000	0.472	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	84332	188	0	129	-1
N.S.	1	1.00	0.61	1.63	788.15	1.76	0.00	1.21	-0.01
time (sec)	N/A	0.043	0.131	0.098	23.976	0.386	0.000	0.506	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.071	0.043	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.051	0.042	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	69	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.048	0.041	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.041	0.040	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.040	0.040	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.048	0.041	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.050	0.059	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.059	0.062	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.066	0.050	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	74	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.057	0.051	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	18	24	26	41	30	38
N.S.	1	1.00	0.65	0.58	0.77	0.84	1.32	0.97	1.23
time (sec)	N/A	0.010	0.020	0.058	0.539	0.363	0.340	0.466	0.428

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	46	68	59	187	59	67
N.S.	1	1.00	0.77	0.82	1.21	1.05	3.34	1.05	1.20
time (sec)	N/A	0.025	0.067	0.065	0.486	0.364	0.856	0.433	0.326

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	62	111	90	359	75	96
N.S.	1	1.00	0.69	0.77	1.37	1.11	4.43	0.93	1.19
time (sec)	N/A	0.042	0.139	0.079	0.550	0.387	1.562	0.511	0.385

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	75	151	121	592	88	96
N.S.	1	1.00	0.62	0.71	1.42	1.14	5.58	0.83	0.91
time (sec)	N/A	0.063	0.186	0.095	0.488	0.410	3.259	0.427	0.515

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	20	18	24	26	41	30	38
N.S.	1	1.00	0.61	0.55	0.73	0.79	1.24	0.91	1.15
time (sec)	N/A	0.009	0.021	0.054	0.503	0.378	0.338	0.426	0.294

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	43	46	69	59	192	61	67
N.S.	1	1.00	0.74	0.79	1.19	1.02	3.31	1.05	1.16
time (sec)	N/A	0.024	0.057	0.072	0.498	0.369	0.706	0.440	0.309

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	64	112	90	364	77	95
N.S.	1	1.00	0.78	0.77	1.35	1.08	4.39	0.93	1.14
time (sec)	N/A	0.042	0.098	0.079	0.513	0.358	1.443	0.452	0.373

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	66	77	152	121	597	90	97
N.S.	1	1.00	0.61	0.71	1.41	1.12	5.53	0.83	0.90
time (sec)	N/A	0.066	0.150	0.098	0.491	0.369	3.167	0.486	0.491

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	20	18	24	26	42	30	38
N.S.	1	1.00	0.61	0.55	0.73	0.79	1.27	0.91	1.15
time (sec)	N/A	0.009	0.019	0.050	0.499	0.364	0.326	0.425	0.254

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	43	46	69	59	192	61	67
N.S.	1	1.00	0.74	0.79	1.19	1.02	3.31	1.05	1.16
time (sec)	N/A	0.021	0.019	0.060	0.483	0.363	0.733	0.468	0.002

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	64	112	90	366	77	96
N.S.	1	1.00	0.78	0.77	1.35	1.08	4.41	0.93	1.16
time (sec)	N/A	0.041	0.071	0.078	0.510	0.364	1.468	0.437	0.359

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	66	77	152	121	597	90	97
N.S.	1	1.00	0.61	0.71	1.41	1.12	5.53	0.83	0.90
time (sec)	N/A	0.063	0.020	0.088	0.494	0.391	3.128	0.446	0.002

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	18	24	26	44	30	38
N.S.	1	1.00	0.65	0.58	0.77	0.84	1.42	0.97	1.23
time (sec)	N/A	0.009	0.019	0.050	0.483	0.377	0.343	0.427	0.255

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	46	68	59	190	59	67
N.S.	1	1.00	0.77	0.82	1.21	1.05	3.39	1.05	1.20
time (sec)	N/A	0.020	0.018	0.063	0.488	0.352	0.880	0.440	0.002

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	65	62	111	90	362	75	95
N.S.	1	1.00	0.80	0.77	1.37	1.11	4.47	0.93	1.17
time (sec)	N/A	0.041	0.073	0.076	0.589	0.380	1.555	0.440	0.358

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	75	151	121	595	88	96
N.S.	1	1.00	0.62	0.71	1.42	1.14	5.61	0.83	0.91
time (sec)	N/A	0.063	0.020	0.090	0.501	0.359	3.281	0.442	0.002

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	34	48	46	41	34	17
N.S.	1	1.00	1.00	0.52	0.74	0.71	0.63	0.52	0.26
time (sec)	N/A	0.013	0.019	0.051	0.279	0.354	0.325	0.409	0.361

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	143	64	91	88	228	62	47
N.S.	1	1.00	1.59	0.71	1.01	0.98	2.53	0.69	0.52
time (sec)	N/A	0.029	0.060	0.065	0.284	0.376	0.667	0.461	0.320

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	217	94	135	129	474	78	76
N.S.	1	1.00	1.89	0.82	1.17	1.12	4.12	0.68	0.66
time (sec)	N/A	0.051	0.133	0.078	0.288	0.364	1.243	0.541	0.674

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	296	124	174	170	813	91	102
N.S.	1	1.00	2.11	0.89	1.24	1.21	5.81	0.65	0.73
time (sec)	N/A	0.078	0.199	0.096	0.299	0.381	2.757	0.501	1.969

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	38	50	46	44	38	17
N.S.	1	1.00	1.00	0.60	0.79	0.73	0.70	0.60	0.27
time (sec)	N/A	0.013	0.019	0.051	0.283	0.377	0.355	0.442	0.332

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	139	72	94	88	240	68	47
N.S.	1	1.00	1.58	0.82	1.07	1.00	2.73	0.77	0.53
time (sec)	N/A	0.028	0.057	0.069	0.304	0.373	0.696	0.522	0.343

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	211	106	137	129	490	84	76
N.S.	1	1.00	1.87	0.94	1.21	1.14	4.34	0.74	0.67
time (sec)	N/A	0.051	0.101	0.078	0.295	0.406	1.264	0.457	0.673

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	288	140	177	170	831	97	102
N.S.	1	1.00	2.09	1.01	1.28	1.23	6.02	0.70	0.74
time (sec)	N/A	0.076	0.160	0.098	0.278	0.374	2.736	0.430	2.013

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	38	50	46	44	38	17
N.S.	1	1.00	1.00	0.60	0.79	0.73	0.70	0.60	0.27
time (sec)	N/A	0.013	0.016	0.049	0.284	0.372	0.333	0.491	0.261

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	139	72	94	88	240	68	47
N.S.	1	1.00	1.58	0.82	1.07	1.00	2.73	0.77	0.53
time (sec)	N/A	0.027	0.017	0.060	0.270	0.382	0.687	0.448	0.002

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	211	106	137	129	490	84	75
N.S.	1	1.00	1.87	0.94	1.21	1.14	4.34	0.74	0.66
time (sec)	N/A	0.051	0.089	0.076	0.273	0.385	1.272	0.423	0.650

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	288	140	177	170	831	97	102
N.S.	1	1.00	2.09	1.01	1.28	1.23	6.02	0.70	0.74
time (sec)	N/A	0.072	0.019	0.090	0.286	0.406	2.786	0.448	0.002

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	34	48	46	42	34	17
N.S.	1	1.00	1.00	0.52	0.74	0.71	0.65	0.52	0.26
time (sec)	N/A	0.012	0.018	0.050	0.302	0.377	0.333	0.412	0.262

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	143	64	91	88	231	62	47
N.S.	1	1.00	1.59	0.71	1.01	0.98	2.57	0.69	0.52
time (sec)	N/A	0.027	0.017	0.065	0.290	0.351	0.687	0.462	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	217	94	135	129	478	78	75
N.S.	1	1.00	1.89	0.82	1.17	1.12	4.16	0.68	0.65
time (sec)	N/A	0.049	0.072	0.074	0.274	0.367	1.265	0.450	0.669

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	296	124	174	170	816	91	102
N.S.	1	1.00	2.11	0.89	1.24	1.21	5.83	0.65	0.73
time (sec)	N/A	0.075	0.018	0.092	0.275	0.373	2.689	0.462	0.002

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	177	662	0	437	0	0	-1
N.S.	1	1.00	0.90	3.36	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.546	0.166	0.000	0.152	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	450	0	398	0	0	-1
N.S.	1	1.00	0.85	2.87	0.00	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.367	0.115	0.000	0.129	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	170	0	355	0	0	-1
N.S.	1	1.00	1.00	2.98	0.00	6.23	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.050	0.257	0.000	0.132	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	75	0	146	0	0	52
N.S.	1	1.00	1.00	1.32	0.00	2.56	0.00	0.00	0.91
time (sec)	N/A	0.027	0.037	0.135	0.000	0.104	0.000	0.000	0.390

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	83	217	0	482	0	0	-1
N.S.	1	1.00	0.78	2.05	0.00	4.55	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.143	0.122	0.000	0.141	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	158	489	0	692	0	0	-1
N.S.	1	1.00	0.71	2.21	0.00	3.13	0.00	0.00	-0.00
time (sec)	N/A	0.162	0.622	0.205	0.000	0.151	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	246	0	0	0	0	0	-1
N.S.	1	1.00	2.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	1.269	0.039	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	118	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.154	0.034	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	118	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.127	0.035	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	118	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.125	0.033	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	116	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.125	0.034	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	268	0	0	0	0	0	-1
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	1.374	0.034	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	121	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.162	0.050	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1] had the largest ratio of [14]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	2	1.00	14	0.143
2	A	3	2	1.00	14	0.143
3	A	2	2	1.00	14	0.143
4	A	1	1	1.00	14	0.071
5	A	2	2	1.00	14	0.143
6	A	3	3	1.00	14	0.214
7	A	4	3	1.00	14	0.214
8	A	2	2	1.00	14	0.143
9	A	2	2	1.00	14	0.143
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	14	0.143
12	A	2	2	1.00	14	0.143
13	A	2	2	1.00	14	0.143
14	A	2	2	1.00	12	0.167
15	A	2	2	1.00	13	0.154
16	A	1	1	1.00	12	0.083
17	A	1	1	1.00	12	0.083
18	A	1	1	1.00	12	0.083
19	A	3	3	1.00	12	0.250
20	A	4	4	1.00	12	0.333
21	A	5	4	1.00	12	0.333
22	A	1	1	1.00	12	0.083
23	A	3	3	1.00	12	0.250
24	A	4	4	1.00	12	0.333
25	A	5	4	1.00	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	1	1	1.00	12	0.083
27	A	3	3	1.00	12	0.250
28	A	4	4	1.00	12	0.333
29	A	5	4	1.00	12	0.333
30	A	1	1	1.00	12	0.083
31	A	3	3	1.00	12	0.250
32	A	4	4	1.00	12	0.333
33	A	5	4	1.00	12	0.333
34	A	2	2	1.00	12	0.167
35	A	4	4	1.00	12	0.333
36	A	5	5	1.00	12	0.417
37	A	6	5	1.00	12	0.417
38	A	2	2	1.00	12	0.167
39	A	4	4	1.00	12	0.333
40	A	5	5	1.00	12	0.417
41	A	6	5	1.00	12	0.417
42	A	2	2	1.00	12	0.167
43	A	4	4	1.00	12	0.333
44	A	5	5	1.00	12	0.417
45	A	6	5	1.00	12	0.417
46	A	2	2	1.00	12	0.167
47	A	4	4	1.00	12	0.333
48	A	5	5	1.00	12	0.417
49	A	6	5	1.00	12	0.417
50	A	7	7	1.00	14	0.500
51	A	6	6	1.00	14	0.429
52	A	2	2	1.00	14	0.143
53	A	2	2	1.00	14	0.143
54	A	4	4	1.00	14	0.286
55	A	7	7	1.00	14	0.500
56	A	3	3	1.00	14	0.214
57	A	3	3	1.00	14	0.214
58	A	3	3	1.00	14	0.214
59	A	3	3	1.00	14	0.214
60	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	14	0.214
62	A	3	3	1.00	12	0.250

Chapter 3

Listing of integrals

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3.3	$\int (a + a \cos(c + dx))^{3/2} dx$	46
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3.10	$\int \sqrt[3]{a + a \cos(c + dx)} dx$	72
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3.14	$\int (a + a \cos(c + dx))^n dx$	84
3.15	$\int (a - a \cos(c + dx))^n dx$	87
3.16	$\int (2 + 2 \cos(c + dx))^n dx$	90
3.17	$\int (2 - 2 \cos(c + dx))^n dx$	93
3.18	$\int \frac{1}{5 + 3 \cos(c + dx)} dx$	96
3.19	$\int \frac{1}{(5 + 3 \cos(c + dx))^2} dx$	99
3.20	$\int \frac{1}{(5 + 3 \cos(c + dx))^3} dx$	103
3.21	$\int \frac{1}{(5 + 3 \cos(c + dx))^4} dx$	107
3.22	$\int \frac{1}{5 - 3 \cos(c + dx)} dx$	112
3.23	$\int \frac{1}{(5 - 3 \cos(c + dx))^2} dx$	115
3.24	$\int \frac{1}{(5 - 3 \cos(c + dx))^3} dx$	119

3.25	$\int \frac{1}{(5-3 \cos(c+dx))^4} dx$	123
3.26	$\int \frac{1}{-5+3 \cos(c+dx)} dx$	128
3.27	$\int \frac{1}{(-5+3 \cos(c+dx))^2} dx$	131
3.28	$\int \frac{1}{(-5+3 \cos(c+dx))^3} dx$	135
3.29	$\int \frac{1}{(-5+3 \cos(c+dx))^4} dx$	139
3.30	$\int \frac{1}{-5-3 \cos(c+dx)} dx$	144
3.31	$\int \frac{1}{(-5-3 \cos(c+dx))^2} dx$	147
3.32	$\int \frac{1}{(-5-3 \cos(c+dx))^3} dx$	151
3.33	$\int \frac{1}{(-5-3 \cos(c+dx))^4} dx$	155
3.34	$\int \frac{1}{3+5 \cos(c+dx)} dx$	160
3.35	$\int \frac{1}{(3+5 \cos(c+dx))^2} dx$	163
3.36	$\int \frac{1}{(3+5 \cos(c+dx))^3} dx$	167
3.37	$\int \frac{1}{(3+5 \cos(c+dx))^4} dx$	172
3.38	$\int \frac{1}{3-5 \cos(c+dx)} dx$	177
3.39	$\int \frac{1}{(3-5 \cos(c+dx))^2} dx$	180
3.40	$\int \frac{1}{(3-5 \cos(c+dx))^3} dx$	184
3.41	$\int \frac{1}{(3-5 \cos(c+dx))^4} dx$	189
3.42	$\int \frac{1}{-3+5 \cos(c+dx)} dx$	194
3.43	$\int \frac{1}{(-3+5 \cos(c+dx))^2} dx$	197
3.44	$\int \frac{1}{(-3+5 \cos(c+dx))^3} dx$	201
3.45	$\int \frac{1}{(-3+5 \cos(c+dx))^4} dx$	206
3.46	$\int \frac{1}{-3-5 \cos(c+dx)} dx$	211
3.47	$\int \frac{1}{(-3-5 \cos(c+dx))^2} dx$	214
3.48	$\int \frac{1}{(-3-5 \cos(c+dx))^3} dx$	218
3.49	$\int \frac{1}{(-3-5 \cos(c+dx))^4} dx$	223
3.50	$\int (a + b \cos(c + dx))^{5/2} dx$	228
3.51	$\int (a + b \cos(c + dx))^{3/2} dx$	233
3.52	$\int \sqrt{a + b \cos(c + dx)} dx$	238
3.53	$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$	242
3.54	$\int \frac{1}{(a+b \cos(c+dx))^{3/2}} dx$	245
3.55	$\int \frac{1}{(a+b \cos(c+dx))^{5/2}} dx$	249
3.56	$\int (a + b \cos(c + dx))^{4/3} dx$	254
3.57	$\int (a + b \cos(c + dx))^{2/3} dx$	258
3.58	$\int \sqrt[3]{a + b \cos(c + dx)} dx$	262
3.59	$\int \frac{1}{\sqrt[3]{a + b \cos(c + dx)}} dx$	266
3.60	$\int \frac{1}{(a+b \cos(c+dx))^{2/3}} dx$	270
3.61	$\int \frac{1}{(a+b \cos(c+dx))^{4/3}} dx$	274
3.62	$\int (a + b \cos(c + dx))^n dx$	278

3.1 $\int (a + a \cos(c + dx))^{7/2} dx$

Optimal. Leaf size=119

$$\frac{256a^4 \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{64a^3 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} + \frac{24a^2 (a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2a \sin(c + dx) (a + a \cos(c + dx))^{5/2}}{7d}$$

[Out] $24/35*a^2*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+2/7*a*(a+a*\cos(d*x+c))^(5/2)*\sin(d*x+c)/d+256/35*a^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+64/35*a^3*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A]

time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\frac{256a^4 \sin(c + dx)}{35d \sqrt{a \cos(c + dx) + a}} + \frac{64a^3 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{35d} + \frac{24a^2 \sin(c + dx) (a \cos(c + dx) + a)^{3/2}}{35d} + \frac{2a \sin(c + dx) (a \cos(c + dx) + a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^(7/2), x]$

[Out] $(256*a^4*\text{Sin}[c + d*x])/(35*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (64*a^3*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(35*d) + (24*a^2*(a + a*\text{Cos}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(35*d) + (2*a*(a + a*\text{Cos}[c + d*x])^(5/2)*\text{Sin}[c + d*x])/(7*d)$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2726

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]^(n_), x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((a + b*\text{Sin}[c + d*x])^(n - 1)/(d*n)), x] + \text{Dist}[a*((2*n - 1)/n), \text{Int}[(a + b*\text{Sin}[c + d*x])^(n - 1), x], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \cos(c + dx))^{7/2} dx &= \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{1}{7}(12a) \int (a + a \cos(c + dx))^{5/2} dx \\
&= \frac{24a^2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&= \frac{64a^3 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} + \frac{24a^2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} \\
&= \frac{256a^4 \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{64a^3 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} + \frac{24a^2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 83, normalized size = 0.70

$$\frac{a^3 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) (1225 \sin\left(\frac{1}{2}(c + dx)\right) + 245 \sin\left(\frac{3}{2}(c + dx)\right) + 49 \sin\left(\frac{5}{2}(c + dx)\right) + 5 \sin\left(\frac{7}{2}(c + dx)\right))}{140d}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(7/2), x]`

```
[Out] (a^3*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1225*Sin[(c + d*x)/2] + 245*Sin[(3*(c + d*x))/2] + 49*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2])/(140*d)
```

Maple [A]

time = 0.08, size = 86, normalized size = 0.72

method	result	size
default	$ \frac{16a^4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(5 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 6 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 16\right) \sqrt{2}}{35 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d} $	86

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a+a*cos(d*x+c))^(7/2), x, method=_RETURNVERBOSE)`

```
[Out] 16/35*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(5*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/2*c)^4+8*cos(1/2*d*x+1/2*c)^2+16)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]

time = 0.53, size = 77, normalized size = 0.65

$$\frac{\left(5 \sqrt{2} a^3 \sin\left(\frac{7}{2} dx + \frac{7}{2} c\right) + 49 \sqrt{2} a^3 \sin\left(\frac{5}{2} dx + \frac{5}{2} c\right) + 245 \sqrt{2} a^3 \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 1225 \sqrt{2} a^3 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sqrt{a}}{140d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] 1/140*(5*sqrt(2)*a^3*sin(7/2*d*x + 7/2*c) + 49*sqrt(2)*a^3*sin(5/2*d*x + 5/2*c) + 245*sqrt(2)*a^3*sin(3/2*d*x + 3/2*c) + 1225*sqrt(2)*a^3*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Fricas [A]

time = 0.39, size = 75, normalized size = 0.63

$$\frac{2(5a^3 \cos(dx+c)^3 + 27a^3 \cos(dx+c)^2 + 71a^3 \cos(dx+c) + 177a^3) \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{35(d \cos(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 2/35*(5*a^3*cos(d*x + c)^3 + 27*a^3*cos(d*x + c)^2 + 71*a^3*cos(d*x + c) + 177*a^3)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(7/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep

Giac [A]

time = 0.60, size = 108, normalized size = 0.91

$$\frac{\sqrt{2} (5a^3 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{7}{2}dx + \frac{7}{2}c) + 49a^3 \operatorname{sgn}(\cos(\frac{3}{2}dx + \frac{3}{2}c)) \sin(\frac{5}{2}dx + \frac{5}{2}c) + 245a^3 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{3}{2}dx + \frac{3}{2}c) + 1225a^3 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{1}{2}dx + \frac{1}{2}c)) \sqrt{a}}{140d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] 1/140*sqrt(2)*(5*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 49*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 245*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 1225*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cos(c + dx))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(c + d*x))^(7/2),x)

[Out] int((a + a*cos(c + d*x))^(7/2), x)

3.2 $\int (a + a \cos(c + dx))^{5/2} dx$

Optimal. Leaf size=89

$$\frac{64a^3 \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} + \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $2/5*a*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+64/15*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+16/15*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A]

time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\frac{64a^3 \sin(c + dx)}{15d\sqrt{a \cos(c + dx) + a}} + \frac{16a^2 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{15d} + \frac{2a \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^(5/2), x]$

[Out] $(64*a^3*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (16*a^2*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*d) + (2*a*(a + a*\text{Cos}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(5*d)$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2726

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]^(n_), x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((a + b*\text{Sin}[c + d*x])^(n - 1)/(d*n)), x] + \text{Dist}[a*((2*n - 1)/n), \text{Int}[(a + b*\text{Sin}[c + d*x])^(n - 1), x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[n - 1/2, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^{5/2} dx &= \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{1}{5}(8a) \int (a + a \cos(c + dx))^{3/2} dx \\ &= \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \\ &= \frac{64a^3 \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} + \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \end{aligned}$$

Mathematica [A]

time = 0.09, size = 71, normalized size = 0.80

$$\frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(150 \sin\left(\frac{1}{2}(c + dx)\right) + 25 \sin\left(\frac{3}{2}(c + dx)\right) + 3 \sin\left(\frac{5}{2}(c + dx)\right)\right)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(5/2), x]**[Out]** (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(150*Sin[(c + d*x)/2] + 25*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)**Maple [A]**

time = 0.09, size = 73, normalized size = 0.82

method	result	size
default	$\frac{8a^3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(3 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8\right) \sqrt{2}}{15 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	73

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(5/2), x, method=_RETURNVERBOSE)**[Out]** 8/15*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(3*cos(1/2*d*x+1/2*c)^4+4*cos(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d**Maxima [A]**

time = 0.56, size = 60, normalized size = 0.67

$$\frac{\left(3 \sqrt{2} a^2 \sin\left(\frac{5}{2} dx + \frac{5}{2} c\right) + 25 \sqrt{2} a^2 \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 150 \sqrt{2} a^2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sqrt{a}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(5/2), x, algorithm="maxima")**[Out]** 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d**Fricas [A]**

time = 0.35, size = 62, normalized size = 0.70

$$\frac{2 \left(3 a^2 \cos(dx + c)^2 + 14 a^2 \cos(dx + c) + 43 a^2\right) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{15 (d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] $2/15*(3*a^2*\cos(d*x + c)^2 + 14*a^2*\cos(d*x + c) + 43*a^2)*\sqrt{a*\cos(d*x + c) + a}*\sin(d*x + c)/(d*\cos(d*x + c) + d)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(c + dx) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(5/2),x)

[Out] Integral((a*cos(c + d*x) + a)**(5/2), x)

Giac [A]

time = 0.47, size = 84, normalized size = 0.94

$$\frac{\sqrt{2} (3 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 25 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 150 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] $1/30*\sqrt{2}*(3*a^2*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 25*a^2*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) + 150*a^2*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\sqrt{a}/d$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cos(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(c + d*x))^(5/2),x)

[Out] int((a + a*cos(c + d*x))^(5/2), x)

3.3 $\int (a + a \cos(c + dx))^{3/2} dx$

Optimal. Leaf size=59

$$\frac{8a^2 \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $8/3*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/3*a*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\frac{8a^2 \sin(c + dx)}{3d \sqrt{a \cos(c + dx) + a}} + \frac{2a \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}, x]$

[Out] $(8*a^2*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2726

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[a*((2*n-1)/n), \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[n - 1/2, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^{3/2} dx &= \frac{2a \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}(4a) \int \sqrt{a + a \cos(c + dx)} dx \\ &= \frac{8a^2 \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 55, normalized size = 0.93

$$\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(3/2), x]``[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d)`**Maple [A]**

time = 0.08, size = 58, normalized size = 0.98

method	result	size
default	$\frac{4a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right) \sqrt{2}}{3 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	58

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a+a*cos(d*x+c))^(3/2), x, method=_RETURNVERBOSE)``[Out] 4/3*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(cos(1/2*d*x+1/2*c)^2+2)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d`**Maxima [A]**

time = 0.54, size = 38, normalized size = 0.64

$$\frac{\left(\sqrt{2} a \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 9 \sqrt{2} a \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sqrt{a}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+a*cos(d*x+c))^(3/2), x, algorithm="maxima")``[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d`**Fricas [A]**

time = 0.36, size = 44, normalized size = 0.75

$$\frac{2(a \cos(dx + c) + 5a) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{3(d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] $2/3*(a*\cos(d*x + c) + 5*a)*\sqrt{a*\cos(d*x + c) + a}*\sin(d*x + c)/(d*\cos(d*x + c) + d)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(c + dx) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(3/2),x)

[Out] Integral((a*cos(c + d*x) + a)**(3/2), x)

Giac [A]

time = 0.47, size = 55, normalized size = 0.93

$$\frac{\sqrt{2} \left(\operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 9 \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right) \sqrt{a}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] $1/3*\sqrt{2}*(a*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) + 9*a*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\sqrt{a}/d$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a \cos(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(c + d*x))^(3/2),x)

[Out] int((a + a*cos(c + d*x))^(3/2), x)

3.4 $\int \sqrt{a + a \cos(c + dx)} dx$

Optimal. Leaf size=26

$$\frac{2a \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] 2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$\frac{2a \sin(c + dx)}{d \sqrt{a \cos(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2a \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

Mathematica [A]

time = 0.02, size = 29, normalized size = 1.12

$$\frac{2\sqrt{a(1 + \cos(c + dx))} \tan\left(\frac{1}{2}(c + dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d

Maple [A]

time = 0.00, size = 43, normalized size = 1.65

method	result	size
default	$\frac{2a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2}}{\sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	43
risch	$-\frac{i\sqrt{2} \sqrt{a \left(e^{i(dx+c)} + 1\right)^2 e^{-i(dx+c)} \left(e^{i(dx+c)} - 1\right)}}{\left(e^{i(dx+c)} + 1\right)d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]

time = 0.53, size = 20, normalized size = 0.77

$$\frac{2 \sqrt{2} \sqrt{a} \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A]

time = 0.38, size = 32, normalized size = 1.23

$$\frac{2 \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{d \cos(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cos(c + dx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*cos(c + d*x) + a), x)

Giac [A]

time = 0.49, size = 30, normalized size = 1.15

$$\frac{2 \sqrt{2} \sqrt{a} \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d

Mupad [B]

time = 0.00, size = 33, normalized size = 1.27

$$\frac{2 \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)}}{d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(c + d*x))^(1/2),x)

[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))

$$3.5 \quad \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a + a \cos(c + dx)}} \right)}{\sqrt{a} d}$$

[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2728, 212}

$$\frac{\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c + dx) + a}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx = -\frac{2 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d}$$

$$= \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a + a \cos(c + dx)}}\right)}{\sqrt{a} d}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 0.87

$$\frac{2 \tanh^{-1}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \cos\left(\frac{1}{2}(c + dx)\right)}{d \sqrt{a(1 + \cos(c + dx))}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]``[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.03, size = 54, normalized size = 1.17

method	result	size
default	$\frac{\sqrt{2} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \text{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} 1\right)}{d \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \text{csgn}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$	54

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/d*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^(1/2)/csgn(cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)*InverseJacobiAM(1/2*d*x+1/2*c,1)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 90 vs.

2(37) = 74.

time = 0.54, size = 90, normalized size = 1.96

$$\frac{\sqrt{2} \log\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) - \sqrt{2} \log\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{2 \sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)

Fricas [A]

time = 0.38, size = 126, normalized size = 2.74

$$\left[\frac{\sqrt{2} \log \left(\frac{\cos(dx+c)^2 - 2\sqrt{2} \sqrt{a \cos(dx+c) + a} \sin(dx+c) - 2 \cos(dx+c) - 3}{\sqrt{a} \cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{2\sqrt{a}d}, -\frac{\sqrt{2} \sqrt{-\frac{1}{a}} \arctan \left(\frac{\sqrt{2} \sqrt{a \cos(dx+c) + a} \sqrt{-\frac{1}{a}}}{\sin(dx+c)} \right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a)/sin(d*x + c))/d]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cos(c + dx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x)

[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(37) = 74.

time = 0.49, size = 93, normalized size = 2.02

$$\frac{\sqrt{2} \log \left(\left| \frac{1}{\sin(\frac{1}{2} dx + \frac{1}{2} c)} + \sin(\frac{1}{2} dx + \frac{1}{2} c) + 2 \right| \right)}{\sqrt{a} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{\sqrt{2} \log \left(\left| \frac{1}{\sin(\frac{1}{2} dx + \frac{1}{2} c)} + \sin(\frac{1}{2} dx + \frac{1}{2} c) - 2 \right| \right)}{\sqrt{a} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2)))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c)))/d

Mupad [B]

time = 0.00, size = 45, normalized size = 0.98

$$\frac{F\left(\frac{c}{2} + \frac{dx}{2} \mid 1\right) \sqrt{\frac{2(a + a \cos(c + dx))}{a}}}{d \sqrt{a + a \cos(c + dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a*cos(c + d*x))^(1/2),x)

[Out] (ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*cos(c + d*x))^(1/2))

3.6 $\int \frac{1}{(a+a \cos(c+dx))^{3/2}} dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}}$$

[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx) + a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{\sin(c+dx)}{2d(a \cos(c+dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(-3/2),x]

[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx}{4a} \\
&= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{2ad} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a + a \cos(c + dx)}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 63, normalized size = 0.82

$$\frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \left(\tanh^{-1}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \cos\left(\frac{1}{2}(c + dx)\right) + \tan\left(\frac{1}{2}(c + dx)\right)\right)}{d(a(1 + \cos(c + dx)))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(-3/2), x]``[Out] (Cos[(c + d*x)/2]^2*(ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + Tan[(c + d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2))`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(62) = 124.

time = 0.09, size = 138, normalized size = 1.79

method	result
default	$ \frac{\sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)}{4a^{\frac{5}{2}} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d} $

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+a*cos(d*x+c))^(3/2), x, method=_RETURNVERBOSE)`

```
[Out] 1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1
/2*d*x+1/2*c)^2+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(62) = 124$.

time = 1.16, size = 15721, normalized size = 204.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{4} \cdot (32 \cdot (\cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(2d*x + 2c) + \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + \cos(d*x + c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \cos(3d*x + 3c)^2 + 96 \cdot (\cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(3d*x + 3c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(2d*x + 2c) - (3 \cdot \cos(d*x + c) + 1) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - \cos(3d*x + 3c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - 3 \cdot \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \cos(\frac{4}{3} \arctan2(\sin(\frac{3}{2}d*x + \frac{3}{2}c), \cos(\frac{3}{2}d*x + \frac{3}{2}c)))^2 + 96 \cdot (\cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(3d*x + 3c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(2d*x + 2c) - (3 \cdot \cos(d*x + c) + 1) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - \cos(3d*x + 3c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - 3 \cdot \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \cos(\frac{2}{3} \arctan2(\sin(\frac{3}{2}d*x + \frac{3}{2}c), \cos(\frac{3}{2}d*x + \frac{3}{2}c)))^2 - 32 \cdot (\cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(2d*x + 2c) + \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + \cos(d*x + c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \sin(3d*x + 3c)^2 + 32 \cdot (6 \cdot \cos(d*x + c) + 1) \cdot \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 9 \cdot \cos(2d*x + 2c)^2 \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 96 \cdot \sin(2d*x + 2c)^2 \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 96 \cdot (\cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(3d*x + 3c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(2d*x + 2c) - (3 \cdot \cos(d*x + c) + 1) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - \cos(3d*x + 3c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - 3 \cdot \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \sin(\frac{4}{3} \arctan2(\sin(\frac{3}{2}d*x + \frac{3}{2}c), \cos(\frac{3}{2}d*x + \frac{3}{2}c)))^2 + 96 \cdot (\cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(3d*x + 3c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(2d*x + 2c) - (3 \cdot \cos(d*x + c) + 1) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - \cos(3d*x + 3c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) - 3 \cdot \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 3 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \sin(\frac{2}{3} \arctan2(\sin(\frac{3}{2}d*x + \frac{3}{2}c), \cos(\frac{3}{2}d*x + \frac{3}{2}c)))^2 + 32 \cdot (2 \cdot (3 \cdot \cos(d*x + c) + 1) \cdot \cos(2d*x + 2c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 3 \cdot \cos(2d*x + 2c)^2 \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 3 \cdot \sin(2d*x + 2c)^2 \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 2 \cdot (3 \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c)) \cdot \sin(2d*x + 2c) + (3 \cdot \cos(d*x + c))^2 + 3 \cdot \sin(d*x + c)^2 + 2 \cdot \cos(d*x + c) \cdot \sin(\frac{3}{2}d*x + \frac{3}{2}c) + 2 \cdot \cos(\frac{3}{2}d*x + \frac{3}{2}c) \cdot \sin(d*x + c)) \cdot \cos(3d*x + 3c) - 4 \cdot (6 \cdot (\sin(2d*x + 2c) + \sin(d*x + c)) \cdot \sin(3d*x + 3c)^2 + \sin(3d*x + 3c)^3 + (2 \cdot (3 \cdot \cos(2d*x + 2c) + 3 \cdot \cos(d*x + c) + 1) \cdot \cos(3d*x + 3c) + \cos(3d*x + 3c)^2 + 6 \cdot (3 \cdot \cos(d*x + c) + 1) \cdot \cos(2d*x + 2c) + 9 \cdot \cos(2d*x + 2c)^2 + 9 \cdot \cos(d*x + c)^2 + 9 \cdot \sin(2d*x + 2c)^2 + 18 \cdot \sin(2d*x + 2c) \cdot \sin(d*x + c) + 9 \cdot \sin(d*x + c)^2 + 6 \cdot \cos(d*x + c) + 1) \cdot \sin(3d*x + 3c) + 3 \cdot (2 \cdot (3 \cdot \cos(2d*x + 2c) + 3 \cdot \cos(d*x + c) + 1) \cdot \cos(3d*x + 3c) + \cos(3d*x + 3c)^2 + 6 \cdot (3 \cdot \cos(d*x + c) + 1) \cdot \cos(2d*x + 2c) + 9 \cdot \cos(2d*x + 2c)^2 + 9 \cdot \cos(d*x + c)^2 + 6 \cdot (\sin(2d*x + 2c) + \sin$

```

(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 1
8*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*cos(2*
d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*
(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x +
c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*
c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
+ c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*si
n(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*c
) - 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(3
/2*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*d
*x + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 48*(cos
(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c
) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*
x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*
c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 16*(cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) + 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))
*sin(3*d*x + 3*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x +
3/2*c))*sin(2*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*sin
(3/2*d*x + 3/2*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d*
x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)
^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c
)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x +
c)^2 + 6*cos(d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
- 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*...

```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(62) = 124.

time = 0.37, size = 153, normalized size = 1.99

$$\frac{\sqrt{2} (\cos(dx+c)^2 + 2 \cos(dx+c) + 1) \sqrt{a} \log\left(\frac{-a \cos(dx+c)^2 - 2\sqrt{2} \sqrt{a \cos(dx+c) + a} \sqrt{a} \sin(dx+c) - 2a \cos(dx+c) - 3a}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1}\right) + 4 \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{8 (a^2 d \cos(dx+c)^2 + 2 a^2 d \cos(dx+c) + a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x +

c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(c + dx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))**(3/2),x)

[Out] Integral((a*cos(c + d*x) + a)**(-3/2), x)

Giac [A]

time = 0.47, size = 112, normalized size = 1.45

$$\frac{\sqrt{2} \left(\frac{\log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{\log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{2 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1) \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} \right)}{8 \sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/8*sqrt(2)*(log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - log(-sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - 2*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(1/2*d*x + 1/2*c))))/(sqrt(a)*d)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a*cos(c + d*x))^(3/2),x)

[Out] int(1/(a + a*cos(c + d*x))^(3/2), x)

3.7 $\int \frac{1}{(a+a \cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=107

$$\frac{3 \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a + a \cos(c+dx)}} \right)}{16\sqrt{2} a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} + \frac{3 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$\frac{3 \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx) + a}} \right)}{16\sqrt{2} a^{5/2}d} + \frac{3 \sin(c+dx)}{16ad(a \cos(c+dx) + a)^{3/2}} + \frac{\sin(c+dx)}{4d(a \cos(c+dx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(-5/2), x]

[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx}{8a} \\
&= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx}{32a^2} \\
&= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} - \frac{3 \text{Subst}\left(\int \frac{1}{2a - x^2} dx\right)}{32a^2} \\
&= \frac{3 \tanh^{-1}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \cos(c + dx)}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 65, normalized size = 0.61

$$\frac{24 \tanh^{-1}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \cos^5\left(\frac{1}{2}(c + dx)\right) + 14 \sin(c + dx) + 3 \sin(2(c + dx))}{32d(a(1 + \cos(c + dx)))^{5/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(-5/2), x]`

```
[Out] (24*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 14*Sin[c + d*x] + 3*Sin[2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A]

time = 0.10, size = 174, normalized size = 1.63

method	result
default	$ \frac{\sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(3\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 3\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)}{32a^{7/2} \cos\left(\frac{dx}{2} + \frac{c}{2}\right)^3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} $

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+a*cos(d*x+c))^(5/2), x, method=_RETURNVERBOSE)`

```
[Out] 1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a
```

$$\cos(1/2*d*x+1/2*c)^4+3*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)^2*a^{(1/2)}+2*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(88) = 176$.

time = 23.98, size = 84332, normalized size = 788.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] $1/32*(512*((2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 2*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) + (2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 2*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(5*d*x + 5*c)^2 + 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 - 512*((2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 2*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) + (2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 2*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(5*d*x +$

[Out] $\frac{1}{64} \cdot (3\sqrt{2} \cdot (\cos(dx + c))^3 + 3\cos(dx + c)^2 + 3\cos(dx + c) + 1) \cdot \sqrt{a} \cdot \log(-a\cos(dx + c)^2 - 2\sqrt{2}\sqrt{a\cos(dx + c) + a}) \cdot \sqrt{a} \cdot \sin(dx + c) - 2a\cos(dx + c) - 3a) / (\cos(dx + c)^2 + 2\cos(dx + c) + 1) + 4\sqrt{2}\sqrt{a\cos(dx + c) + a} \cdot (3\cos(dx + c) + 7) \cdot \sin(dx + c) / (a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(c + dx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))**(5/2),x)

[Out] Integral((a*cos(c + d*x) + a)**(-5/2), x)

Giac [A]

time = 0.51, size = 129, normalized size = 1.21

$$\frac{\sqrt{2} \left(\frac{3 \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{3 \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{2 (3 \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 5 \sin(\frac{1}{2} dx + \frac{1}{2} c))}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} \right)}{64 \sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] $\frac{1}{64} \cdot \sqrt{2} \cdot (3 \cdot \log(\sin(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) / (a^2 \cdot \operatorname{sgn}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c))) - 3 \cdot \log(-\sin(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) / (a^2 \cdot \operatorname{sgn}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c)))) - 2 \cdot (3 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 5 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)) / ((\sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 - 1)^2 \cdot a^2 \cdot \operatorname{sgn}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c)))) / (\sqrt{a} \cdot d)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(1/(a + a*cos(c + d*x))^(5/2), x)

3.8 $\int (a + a \cos(c + dx))^{4/3} dx$

Optimal. Leaf size=67

$$\frac{2 \cdot 2^{5/6} a \sqrt[3]{a + a \cos(c + dx)} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d(1 + \cos(c + dx))^{5/6}}$$

[Out] $2 \cdot 2^{5/6} \cdot a \cdot (a + a \cdot \cos(d \cdot x + c))^{1/3} \cdot \text{hypergeom}([-5/6, 1/2], [3/2], 1/2 - 1/2 \cdot \cos(d \cdot x + c)) \cdot \sin(d \cdot x + c) / d / (1 + \cos(d \cdot x + c))^{5/6}$

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$\frac{2 \cdot 2^{5/6} a \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{d(\cos(c + dx) + 1)^{5/6}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \cdot \text{Cos}[c + d \cdot x])^{4/3}, x]$

[Out] $(2 \cdot 2^{5/6} \cdot a \cdot (a + a \cdot \text{Cos}[c + d \cdot x])^{1/3} \cdot \text{Hypergeometric2F1}[-5/6, 1/2, 3/2, (1 - \text{Cos}[c + d \cdot x])/2] \cdot \text{Sin}[c + d \cdot x]) / (d \cdot (1 + \text{Cos}[c + d \cdot x])^{5/6})$

Rule 2730

$\text{Int}[(a + (b \cdot \sin[(c + d \cdot x)])^{n}), x_Symbol] \rightarrow \text{Simp}[(2^{n+1/2}) \cdot a^{n-1/2} \cdot b \cdot (\text{Cos}[c + d \cdot x] / (d \cdot \text{Sqrt}[a + b \cdot \text{Sin}[c + d \cdot x]])) \cdot \text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2) \cdot (1 - b \cdot (\text{Sin}[c + d \cdot x] / a))], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2731

$\text{Int}[(a + (b \cdot \sin[(c + d \cdot x)])^{n}), x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[n]} \cdot ((a + b \cdot \text{Sin}[c + d \cdot x])^{\text{FracPart}[n]} / (1 + (b/a) \cdot \text{Sin}[c + d \cdot x])^{\text{FracPart}[n]}), \text{Int}[(1 + (b/a) \cdot \text{Sin}[c + d \cdot x])^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^{4/3} dx &= \frac{\left(a \sqrt[3]{a + a \cos(c + dx)}\right) \int (1 + \cos(c + dx))^{4/3} dx}{\sqrt[3]{1 + \cos(c + dx)}} \\ &= \frac{2 \cdot 2^{5/6} a \sqrt[3]{a + a \cos(c + dx)} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d(1 + \cos(c + dx))^{5/6}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 69, normalized size = 1.03

$$\frac{6(a(1 + \cos(c + dx)))^{4/3} \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{11d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(4/3),x]**[Out]** (-6*(a*(1 + Cos[c + d*x]))^(4/3)*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(11*d)**Maple [F]**

time = 0.04, size = 0, normalized size = 0.00

$$\int (a + a \cos(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(4/3),x)**[Out]** int((a+a*cos(d*x+c))^(4/3),x)**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(4/3),x, algorithm="maxima")**[Out]** integrate((a*cos(d*x + c) + a)^(4/3), x)**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(4/3),x, algorithm="fricas")**[Out]** integral((a*cos(d*x + c) + a)^(4/3), x)**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(c + dx) + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(4/3),x)

[Out] Integral((a*cos(c + d*x) + a)**(4/3), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(4/3), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cos(c + dx))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(c + d*x))^(4/3),x)

[Out] int((a + a*cos(c + d*x))^(4/3), x)

3.9 $\int (a + a \cos(c + dx))^{2/3} dx$

Optimal. Leaf size=66

$$\frac{2\sqrt[6]{2} (a + a \cos(c + dx))^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d(1 + \cos(c + dx))^{7/6}}$$

[Out] $2*2^{(1/6)}*(a+a*\cos(d*x+c))^{(2/3)}*\text{hypergeom}([-1/6, 1/2], [3/2], 1/2-1/2*\cos(d*x+c))*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(7/6)}$

Rubi [A]

time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$\frac{2\sqrt[6]{2} \sin(c + dx)(a \cos(c + dx) + a)^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{d(\cos(c + dx) + 1)^{7/6}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(2/3)}, x]$

[Out] $(2*2^{(1/6)}*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Hypergeometric2F1}[-1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(d*(1 + \text{Cos}[c + d*x])^{(7/6)})$

Rule 2730

$\text{Int}[(a + b*\sin[(c + d*x)])^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-2^{(n + 1/2)})*a^{(n - 1/2)}*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]))*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(\text{Sin}[c + d*x]/a))], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rule 2731

$\text{Int}[(a + b*\sin[(c + d*x)])^{(n)}, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[n]}*((a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]}/(1 + (b/a)*\text{Sin}[c + d*x])^{\text{FracPart}[n]}), \text{Int}[(1 + (b/a)*\text{Sin}[c + d*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ !\text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^{2/3} dx &= \frac{(a + a \cos(c + dx))^{2/3} \int (1 + \cos(c + dx))^{2/3} dx}{(1 + \cos(c + dx))^{2/3}} \\ &= \frac{2\sqrt[6]{2} (a + a \cos(c + dx))^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d(1 + \cos(c + dx))^{7/6}} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 69, normalized size = 1.05

$$\frac{6(a(1 + \cos(c + dx)))^{2/3} \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{7d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(2/3),x]**[Out]** (-6*(a*(1 + Cos[c + d*x]))^(2/3)*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(7*d)**Maple [F]**

time = 0.04, size = 0, normalized size = 0.00

$$\int (a + a \cos(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(2/3),x)**[Out]** int((a+a*cos(d*x+c))^(2/3),x)**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")**[Out]** integrate((a*cos(d*x + c) + a)^(2/3), x)**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")**[Out]** integral((a*cos(d*x + c) + a)^(2/3), x)**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(c + dx) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(d*x+c))**(2/3),x)`

[Out] `Integral((a*cos(c + d*x) + a)**(2/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(d*x+c))^(2/3),x, algorithm="giac")`

[Out] `integrate((a*cos(d*x + c) + a)^(2/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a \cos(c + dx))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cos(c + d*x))^(2/3),x)`

[Out] `int((a + a*cos(c + d*x))^(2/3), x)`

3.10 $\int \sqrt[3]{a + a \cos(c + dx)} dx$

Optimal. Leaf size=65

$$\frac{2^{5/6} \sqrt[3]{a + a \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d(1 + \cos(c + dx))^{5/6}}$$

[Out] $2^{(5/6)}*(a+a*\cos(d*x+c))^{(1/3)}*\text{hypergeom}([1/6, 1/2], [3/2], 1/2-1/2*\cos(d*x+c))*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(5/6)}$

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$\frac{2^{5/6} \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{d(\cos(c + dx) + 1)^{5/6}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(1/3)}, x]$

[Out] $(2^{(5/6)}*(a + a*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(d*(1 + \text{Cos}[c + d*x])^{(5/6)})$

Rule 2730

$\text{Int}[(a_ + (b_)*\sin[(c_) + (d_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(-2^{(n + 1/2)})*a^{(n - 1/2)}*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]))*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(\text{Sin}[c + d*x]/a))], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rule 2731

$\text{Int}[(a_ + (b_)*\sin[(c_) + (d_)*(x_)])^{(n_)}, x_Symbol] :> \text{Dist}[a^{\text{IntPart}[n]}*(a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]}/(1 + (b/a)*\text{Sin}[c + d*x])^{\text{FracPart}[n]}], \text{Int}[(1 + (b/a)*\text{Sin}[c + d*x])^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ !\text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int \sqrt[3]{a + a \cos(c + dx)} dx &= \frac{\sqrt[3]{a + a \cos(c + dx)} \int \sqrt[3]{1 + \cos(c + dx)} dx}{\sqrt[3]{1 + \cos(c + dx)}} \\ &= \frac{2^{5/6} \sqrt[3]{a + a \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d(1 + \cos(c + dx))^{5/6}} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 69, normalized size = 1.06

$$\frac{6\sqrt[3]{a(1 + \cos(c + dx))} \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{5d}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(1/3), x]``[Out] (-6*(a*(1 + Cos[c + d*x]))^(1/3)*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(5*d)`**Maple [F]**

time = 0.04, size = 0, normalized size = 0.00

$$\int (a + a \cos(dx + c))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a+a*cos(d*x+c))^(1/3), x)``[Out] int((a+a*cos(d*x+c))^(1/3), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+a*cos(d*x+c))^(1/3), x, algorithm="maxima")``[Out] integrate((a*cos(d*x + c) + a)^(1/3), x)`**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+a*cos(d*x+c))^(1/3), x, algorithm="fricas")``[Out] integral((a*cos(d*x + c) + a)^(1/3), x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a \cos(c + dx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(d*x+c))**(1/3),x)`

[Out] `Integral((a*cos(c + d*x) + a)**(1/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(d*x+c))^(1/3),x, algorithm="giac")`

[Out] `integrate((a*cos(d*x + c) + a)^(1/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a \cos(c + dx))^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cos(c + d*x))^(1/3),x)`

[Out] `int((a + a*cos(c + d*x))^(1/3), x)`

$$3.11 \quad \int \frac{1}{\sqrt[3]{a + a \cos(c + dx)}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt[6]{2} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d \sqrt[6]{1 + \cos(c + dx)} \sqrt[3]{a + a \cos(c + dx)}}$$

[Out] $2^{(1/6)} \text{hypergeom}([1/2, 5/6], [3/2], 1/2 - 1/2 \cos(d*x+c)) \sin(d*x+c) / d / (1 + \cos(d*x+c))^{(1/6)} / (a + a \cos(d*x+c))^{(1/3)}$

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$\frac{\sqrt[6]{2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{d \sqrt[6]{\cos(c + dx) + 1} \sqrt[3]{a \cos(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(-1/3), x]

[Out] $(2^{(1/6)} \text{Hypergeometric2F1}[1/2, 5/6, 3/2, (1 - \text{Cos}[c + d*x])/2] \text{Sin}[c + d*x]) / (d * (1 + \text{Cos}[c + d*x])^{(1/6)} * (a + a \text{Cos}[c + d*x])^{(1/3)})$

Rule 2730

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-2^(n + 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2731

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[a^IntPart[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{a + a \cos(c + dx)}} dx &= \frac{\sqrt[3]{1 + \cos(c + dx)} \int \frac{1}{\sqrt[3]{1 + \cos(c + dx)}} dx}{\sqrt[3]{a + a \cos(c + dx)}} \\ &= \frac{\sqrt[6]{2} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d \sqrt[6]{1 + \cos(c + dx)} \sqrt[3]{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 67, normalized size = 1.03

$$\frac{6 \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{d^3 \sqrt[3]{a(1 + \cos(c + dx))}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(-1/3), x]`

```
[Out] (-6*Cot[(c + d*x)/2]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(d*(a*(1 + Cos[c + d*x]))^(1/3))
```

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+a*cos(d*x+c))^(1/3), x)``[Out] int(1/(a+a*cos(d*x+c))^(1/3), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(1/3), x, algorithm="maxima")``[Out] integrate((a*cos(d*x + c) + a)^(-1/3), x)`**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(1/3), x, algorithm="fricas")``[Out] integral((a*cos(d*x + c) + a)^(-1/3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a \cos(c + dx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))**(1/3),x)``[Out] Integral((a*cos(c + d*x) + a)**(-1/3), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")``[Out] integrate((a*cos(d*x + c) + a)^(-1/3), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + a \cos(c + dx))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a + a*cos(c + d*x))^(1/3),x)``[Out] int(1/(a + a*cos(c + d*x))^(1/3), x)`

$$3.12 \quad \int \frac{1}{(a+a \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt[6]{1 + \cos(c + dx)} \, {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{\sqrt[6]{2} d(a + a \cos(c + dx))^{2/3}}$$

[Out] $1/2*(1+\cos(d*x+c))^{(1/6)}*\text{hypergeom}([1/2, 7/6], [3/2], 1/2-1/2*\cos(d*x+c))*\sin(d*x+c)*2^{(5/6)}/d/(a+a*\cos(d*x+c))^{(2/3)}$

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$\frac{\sin(c + dx) \sqrt[6]{\cos(c + dx) + 1} \, {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{\sqrt[6]{2} d(a \cos(c + dx) + a)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(-2/3), x]

[Out] $((1 + \text{Cos}[c + d*x])^{(1/6)}*\text{Hypergeometric2F1}[1/2, 7/6, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(2^{(1/6)}*d*(a + a*\text{Cos}[c + d*x])^{(2/3)})$

Rule 2730

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n + 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]))]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2731

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \cos(c + dx))^{2/3}} dx &= \frac{(1 + \cos(c + dx))^{2/3} \int \frac{1}{(1 + \cos(c + dx))^{2/3}} dx}{(a + a \cos(c + dx))^{2/3}} \\ &= \frac{\sqrt[6]{1 + \cos(c + dx)} \, {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{\sqrt[6]{2} d(a + a \cos(c + dx))^{2/3}} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 67, normalized size = 1.03

$$\frac{6 \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{d(a(1 + \cos(c + dx)))^{2/3}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Cos[c + d*x])^(-2/3), x]``[Out] (6*Cot[(c + d*x)/2]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(d*(a*(1 + Cos[c + d*x]))^(2/3))`**Maple [F]**

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a \cos(dx + c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+a*cos(d*x+c))^(2/3), x)``[Out] int(1/(a+a*cos(d*x+c))^(2/3), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(2/3), x, algorithm="maxima")``[Out] integrate((a*cos(d*x + c) + a)^(-2/3), x)`**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(2/3), x, algorithm="fricas")``[Out] integral((a*cos(d*x + c) + a)^(-2/3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(c + dx) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))**(2/3),x)``[Out] Integral((a*cos(c + d*x) + a)**(-2/3), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")``[Out] integrate((a*cos(d*x + c) + a)^(-2/3), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + a \cos(c + dx))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a + a*cos(c + d*x))^(2/3),x)``[Out] int(1/(a + a*cos(c + d*x))^(2/3), x)`

$$3.13 \quad \int \frac{1}{(a+a \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=68

$$\frac{{}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{2^{5/6} a d \sqrt[6]{1 + \cos(c + dx)} \sqrt[3]{a + a \cos(c + dx)}}$$

[Out] 1/2*hypergeom([1/2, 11/6], [3/2], 1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/a/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)

Rubi [A]

time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$\frac{\sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2^{5/6} a d \sqrt[6]{\cos(c + dx) + 1} \sqrt[3]{a \cos(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(-4/3), x]

[Out] (Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*a*d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))

Rule 2730

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n + 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2731

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \cos(c + dx))^{4/3}} dx &= \frac{\sqrt[3]{1 + \cos(c + dx)} \int \frac{1}{(1 + \cos(c + dx))^{4/3}} dx}{a \sqrt[3]{a + a \cos(c + dx)}} \\ &= \frac{{}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{2^{5/6} a d \sqrt[6]{1 + \cos(c + dx)} \sqrt[3]{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 69, normalized size = 1.01

$$\frac{6 \cot\left(\frac{1}{2}(c+dx)\right) {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2\left(\frac{1}{2}(c+dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}{5d(a(1+\cos(c+dx)))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(-4/3), x]

[Out] (6*Cot[(c + d*x)/2]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(5*d*(a*(1 + Cos[c + d*x]))^(4/3))

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a \cos(dx + c))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*cos(d*x+c))^(4/3), x)

[Out] int(1/(a+a*cos(d*x+c))^(4/3), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^(-4/3), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((a*cos(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(c + dx) + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))**(4/3),x)``[Out] Integral((a*cos(c + d*x) + a)**(-4/3), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+a*cos(d*x+c))^(4/3),x, algorithm="giac")``[Out] integrate((a*cos(d*x + c) + a)^(-4/3), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a \cos(c + dx))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a + a*cos(c + d*x))^(4/3),x)``[Out] int(1/(a + a*cos(c + d*x))^(4/3), x)`

3.14 $\int (a + a \cos(c + dx))^n dx$

Optimal. Leaf size=73

$$\frac{2^{\frac{1}{2}+n}(1 + \cos(c + dx))^{-\frac{1}{2}-n}(a + a \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d}$$

[Out] $2^{(1/2+n)}(1+\cos(d*x+c))^{(-1/2-n)}(a+a*\cos(d*x+c))^n*\text{hypergeom}([1/2, 1/2-n], [3/2], 1/2-1/2*\cos(d*x+c))*\sin(d*x+c)/d$

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2731, 2730}

$$\frac{2^{n+\frac{1}{2}} \sin(c + dx)(\cos(c + dx) + 1)^{-n-\frac{1}{2}}(a \cos(c + dx) + a)^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^n, x]$

[Out] $(2^{(1/2 + n)}(1 + \text{Cos}[c + d*x])^{(-1/2 - n)}(a + a*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/d$

Rule 2730

$\text{Int}[(a + (b_*)*\sin[(c_*) + (d_*)*(x_*)])^{(n_*)}, x_Symbol] :> \text{Simp}[(-2^{(n + 1/2)})*a^{(n - 1/2)}*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])]*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(\text{Sin}[c + d*x]/a))], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rule 2731

$\text{Int}[(a + (b_*)*\sin[(c_*) + (d_*)*(x_*)])^{(n_*)}, x_Symbol] :> \text{Dist}[a^{\text{IntPart}[n]}*((a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]} / (1 + (b/a)*\text{Sin}[c + d*x])^{\text{FracPart}[n]}), \text{Int}[(1 + (b/a)*\text{Sin}[c + d*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n] \&\& !\text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^n dx &= ((1 + \cos(c + dx))^{-n}(a + a \cos(c + dx))^n) \int (1 + \cos(c + dx))^n dx \\ &= \frac{2^{\frac{1}{2}+n}(1 + \cos(c + dx))^{-\frac{1}{2}-n}(a + a \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 74, normalized size = 1.01

$$\frac{2(a(1 + \cos(c + dx)))^n \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + n; \frac{3}{2} + n; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{d + 2dn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^n,x]**[Out]** (-2*(a*(1 + Cos[c + d*x]))^n*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(d + 2*d*n)**Maple [F]**

time = 0.06, size = 0, normalized size = 0.00

$$\int (a + a \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^n,x)**[Out]** int((a+a*cos(d*x+c))^n,x)**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^n,x, algorithm="maxima")**[Out]** integrate((a*cos(d*x + c) + a)^n, x)**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^n,x, algorithm="fricas")**[Out]** integral((a*cos(d*x + c) + a)^n, x)**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(c + dx) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(d*x+c))**n,x)`

[Out] `Integral((a*cos(c + d*x) + a)**n, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(d*x+c))^n,x, algorithm="giac")`

[Out] `integrate((a*cos(d*x + c) + a)^n, x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cos(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cos(c + d*x))^n,x)`

[Out] `int((a + a*cos(c + d*x))^n, x)`

3.15 $\int (a - a \cos(c + dx))^n dx$

Optimal. Leaf size=75

$$\frac{2^{\frac{1}{2}+n}(1 - \cos(c + dx))^{-\frac{1}{2}-n}(a - a \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 + \cos(c + dx))\right) \sin(c + dx)}{d}$$

[Out] $-2^{(1/2+n)}(1-\cos(d*x+c))^{(-1/2-n)}(a-a*\cos(d*x+c))^n*\text{hypergeom}([1/2, 1/2-n], [3/2], 1/2+1/2*\cos(d*x+c))*\sin(d*x+c)/d$

Rubi [A]

time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2731, 2730}

$$\frac{2^{n+\frac{1}{2}} \sin(c + dx)(1 - \cos(c + dx))^{-n-\frac{1}{2}}(a - a \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(\cos(c + dx) + 1)\right)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a - a*\text{Cos}[c + d*x])^n, x]$

[Out] $-((2^{(1/2 + n)}(1 - \text{Cos}[c + d*x])^{(-1/2 - n)}(a - a*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1 + \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/d$

Rule 2730

$\text{Int}[(a + b*\sin[(c + d*x)])^n, x_Symbol] \rightarrow \text{Simp}[(-2^{(n + 1/2)})*a^{(n - 1/2)}*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]))*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(\text{Sin}[c + d*x]/a))], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rule 2731

$\text{Int}[(a + b*\sin[(c + d*x)])^n, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[n]}*((a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]} / (1 + (b/a)*\text{Sin}[c + d*x])^{\text{FracPart}[n]}), \text{Int}[(1 + (b/a)*\text{Sin}[c + d*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ !\text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int (a - a \cos(c + dx))^n dx &= ((1 - \cos(c + dx))^{-n}(a - a \cos(c + dx))^n) \int (1 - \cos(c + dx))^n dx \\ &= -\frac{2^{\frac{1}{2}+n}(1 - \cos(c + dx))^{-\frac{1}{2}-n}(a - a \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 + \cos(c + dx))\right) \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 75, normalized size = 1.00

$$\frac{\sqrt{2} \sqrt{1 + \cos(c + dx)} (a - a \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + n; \frac{3}{2} + n; \sin^2\left(\frac{1}{2}(c + dx)\right)\right) \tan\left(\frac{1}{2}(c + dx)\right)}{d + 2dn}$$

Antiderivative was successfully verified.

[In] Integrate[(a - a*Cos[c + d*x])^n,x]

[Out] (Sqrt[2]*Sqrt[1 + Cos[c + d*x]]*(a - a*Cos[c + d*x])^n*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, Sin[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(d + 2*d*n)

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int (a - a \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*cos(d*x+c))^n,x)

[Out] int((a-a*cos(d*x+c))^n,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*cos(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((-a*cos(d*x + c) + a)^n, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*cos(d*x+c))^n,x, algorithm="fricas")

[Out] integral((-a*cos(d*x + c) + a)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (-a \cos(c + dx) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*cos(d*x+c))**n,x)

[Out] Integral((-a*cos(c + d*x) + a)**n, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*cos(d*x+c))^n,x, algorithm="giac")

[Out] integrate((-a*cos(d*x + c) + a)^n, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a - a \cos(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a*cos(c + d*x))^n,x)

[Out] int((a - a*cos(c + d*x))^n, x)

3.16 $\int (2 + 2 \cos(c + dx))^n dx$

Optimal. Leaf size=59

$$\frac{2^{\frac{1}{2}+2n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)}}$$

[Out] $2^{(1/2+2*n)} \text{hypergeom}([1/2, 1/2-n], [3/2], 1/2-1/2*\cos(d*x+c)) * \sin(d*x+c) / d / (1+\cos(d*x+c))^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2730}

$$\frac{2^{2n+\frac{1}{2}} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{d \sqrt{\cos(c + dx) + 1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 + 2*\text{Cos}[c + d*x])^n, x]$

[Out] $(2^{(1/2 + 2*n)} \text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1 - \text{Cos}[c + d*x])/2] * \text{Sin}[c + d*x]) / (d * \text{Sqrt}[1 + \text{Cos}[c + d*x]])$

Rule 2730

$\text{Int}[(a + (b_*) * \sin[(c_*) + (d_*) * (x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(-2^{(n + 1/2)}) * a^{(n - 1/2)} * b * (\text{Cos}[c + d*x] / (d * \text{Sqrt}[a + b * \text{Sin}[c + d*x]])) * \text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2) * (1 - b * (\text{Sin}[c + d*x] / a))], x] / ; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rubi steps

$$\int (2 + 2 \cos(c + dx))^n dx = \frac{2^{\frac{1}{2}+2n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)}}$$

Mathematica [A]

time = 0.07, size = 77, normalized size = 1.31

$$\frac{2^{1+n} (1 + \cos(c + dx))^n \cot\left(\frac{1}{2}(c + dx)\right) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + n; \frac{3}{2} + n; \cos^2\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}}{d + 2dn}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*Cos[c + d*x])^n,x]

[Out] -((2^(1 + n)*(1 + Cos[c + d*x])^n*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(d + 2*d*n))

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int (2 + 2 \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+2*cos(d*x+c))^n,x)

[Out] int((2+2*cos(d*x+c))^n,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*cos(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 2)^n, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*cos(d*x+c))^n,x, algorithm="fricas")

[Out] integral((2*cos(d*x + c) + 2)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$2^n \int (\cos(c + dx) + 1)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*cos(d*x+c))**n,x)

[Out] 2**n*Integral((cos(c + d*x) + 1)**n, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*cos(d*x+c))^n,x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 2)^n, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \left(4 \cos \left(\frac{c}{2} + \frac{dx}{2} \right)^2 \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(c + d*x) + 2)^n,x)

[Out] int((4*cos(c/2 + (d*x)/2)^2)^n, x)

3.17 $\int (2 - 2 \cos(c + dx))^n dx$

Optimal. Leaf size=60

$$-\frac{2^{\frac{1}{2}+2n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 + \cos(c + dx))\right) \sin(c + dx)}{d\sqrt{1 - \cos(c + dx)}}$$

[Out] $-2^{(1/2+2*n)}*\text{hypergeom}([1/2, 1/2-n], [3/2], 1/2+1/2*\cos(d*x+c))*\sin(d*x+c)/d/(1-\cos(d*x+c))^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2730}

$$-\frac{2^{2n+\frac{1}{2}} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(\cos(c + dx) + 1)\right)}{d\sqrt{1 - \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 - 2*\text{Cos}[c + d*x])^n, x]$

[Out] $-((2^{(1/2 + 2*n)}*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1 + \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(d*\text{Sqrt}[1 - \text{Cos}[c + d*x]]))$

Rule 2730

$\text{Int}[(a + (b_*)*\sin[(c_*) + (d_*)(x)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(-2^{(n + 1/2)})*a^{(n - 1/2)}*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]))*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(\text{Sin}[c + d*x]/a))], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rubi steps

$$\int (2 - 2 \cos(c + dx))^n dx = -\frac{2^{\frac{1}{2}+2n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - n; \frac{3}{2}; \frac{1}{2}(1 + \cos(c + dx))\right) \sin(c + dx)}{d\sqrt{1 - \cos(c + dx)}}$$

Mathematica [A]

time = 0.06, size = 74, normalized size = 1.23

$$\frac{\sqrt{2} (2 - 2 \cos(c + dx))^n \sqrt{1 + \cos(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + n; \frac{3}{2} + n; \sin^2\left(\frac{1}{2}(c + dx)\right)\right) \tan\left(\frac{1}{2}(c + dx)\right)}{d + 2dn}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 2*Cos[c + d*x])^n,x]

[Out] (Sqrt[2]*(2 - 2*Cos[c + d*x])^n*Sqrt[1 + Cos[c + d*x]]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, Sin[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(d + 2*d*n)

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int (2 - 2 \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-2*cos(d*x+c))^n,x)

[Out] int((2-2*cos(d*x+c))^n,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-2*cos(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((-2*cos(d*x + c) + 2)^n, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-2*cos(d*x+c))^n,x, algorithm="fricas")

[Out] integral((-2*cos(d*x + c) + 2)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (2 - 2 \cos(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-2*cos(d*x+c))^n,x)

[Out] Integral((2 - 2*cos(c + d*x))^n, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-2*cos(d*x+c))^n,x, algorithm="giac")

[Out] integrate((-2*cos(d*x + c) + 2)^n, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \left(4 \sin \left(\frac{c}{2} + \frac{dx}{2} \right)^2 \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2 - 2*cos(c + d*x))^n,x)

[Out] int((4*sin(c/2 + (d*x)/2)^2)^n, x)

$$3.18 \quad \int \frac{1}{5+3 \cos(c+dx)} dx$$

Optimal. Leaf size=31

$$\frac{x}{4} - \frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

[Out] 1/4*x-1/2*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2736}

$$\frac{x}{4} - \frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*Cos[c + d*x])^(-1),x]

[Out] x/4 - ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])]/(2*d)

Rule 2736

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rubi steps

$$\int \frac{1}{5+3 \cos(c+dx)} dx = \frac{x}{4} - \frac{\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 0.65

$$-\frac{\text{ArcTan}\left(2 \cot\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 3*Cos[c + d*x])^(-1),x]

[Out] $-1/2*\text{ArcTan}[2*\text{Cot}[(c + d*x)/2]]/d$

Maple [A]

time = 0.06, size = 18, normalized size = 0.58

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{2d}$	18
risch	$-\frac{i \ln(e^{i(dx+c)} + \frac{1}{3})}{4d} + \frac{i \ln(e^{i(dx+c)} + 3)}{4d}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5+3*cos(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] $1/2*d*\arctan(1/2*\tan(1/2*d*x+1/2*c))$

Maxima [A]

time = 0.54, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*cos(d*x+c)),x, algorithm="maxima")`

[Out] $1/2*\arctan(1/2*\sin(d*x + c)/(\cos(d*x + c) + 1))/d$

Fricas [A]

time = 0.36, size = 26, normalized size = 0.84

$$-\frac{\arctan\left(\frac{5 \cos(dx+c)+3}{4 \sin(dx+c)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*cos(d*x+c)),x, algorithm="fricas")`

[Out] $-1/4*\arctan(1/4*(5*\cos(d*x + c) + 3)/\sin(d*x + c))/d$

Sympy [A]

time = 0.34, size = 41, normalized size = 1.32

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right) + \pi \left\lfloor \frac{c}{\pi} + \frac{dx}{2\pi} - \frac{\pi}{2} \right\rfloor}{2d} & \text{for } d \neq 0 \\ \frac{x}{3 \cos(c) + 5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c)),x)

[Out] Piecewise(((atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(2*d), Ne(d, 0)), (x/(3*cos(c) + 5), True))

Giac [A]

time = 0.47, size = 30, normalized size = 0.97

$$\frac{dx + c - 2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c)),x, algorithm="giac")

[Out] 1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d

Mupad [B]

time = 0.43, size = 38, normalized size = 1.23

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{2d} - \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cos(c + d*x) + 5),x)

[Out] atan(tan(c/2 + (d*x)/2)/2)/(2*d) - (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d)

3.19

$$\int \frac{1}{(5+3\cos(c+dx))^2} dx$$

Optimal. Leaf size=56

$$\frac{5x}{64} - \frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))}$$

[Out] 5/64*x-5/32*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-3/16*sin(d*x+c)/d/(5+3*cos(d*x+c))

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2743, 12, 2736}

$$-\frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{32d} - \frac{3\sin(c+dx)}{16d(3\cos(c+dx)+5)} + \frac{5x}{64}$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*Cos[c + d*x])^(-2), x]

[Out] (5*x)/64 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(32*d) - (3*Sin[c + d*x])/(16*d*(5 + 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2736

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5 + 3 \cos(c + dx))^2} dx &= -\frac{3 \sin(c + dx)}{16d(5 + 3 \cos(c + dx))} - \frac{1}{16} \int -\frac{5}{5 + 3 \cos(c + dx)} dx \\
&= -\frac{3 \sin(c + dx)}{16d(5 + 3 \cos(c + dx))} + \frac{5}{16} \int \frac{1}{5 + 3 \cos(c + dx)} dx \\
&= \frac{5x}{64} - \frac{5 \tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3 \sin(c + dx)}{16d(5 + 3 \cos(c + dx))}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 43, normalized size = 0.77

$$\frac{5 \operatorname{ArcTan}\left(2 \cot\left(\frac{1}{2}(c + dx)\right)\right) + \frac{6 \sin(c+dx)}{5+3 \cos(c+dx)}}{32d}$$

Antiderivative was successfully verified.

`[In] Integrate[(5 + 3*Cos[c + d*x])^(-2), x]``[Out] -1/32*(5*ArcTan[2*Cot[(c + d*x)/2]] + (6*Sin[c + d*x])/(5 + 3*Cos[c + d*x]))/d`**Maple [A]**

time = 0.06, size = 46, normalized size = 0.82

method	result	size
derivativedivides	$ -\frac{\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)} + \frac{5 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{32}}{d} $	46
default	$ -\frac{\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)} + \frac{5 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{32}}{d} $	46
risch	$ -\frac{i(5 e^{i(dx+c)} + 3)}{8d(3 e^{2i(dx+c)} + 10 e^{i(dx+c)} + 3)} + \frac{5i \ln(e^{i(dx+c)} + 3)}{64d} - \frac{5i \ln(e^{i(dx+c)} + \frac{1}{3})}{64d} $	83

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(5+3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)``[Out] 1/d*(-3/16*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+4)+5/32*arctan(1/2*tan(1/2*d*x+1/2*c)))`**Maxima [A]**

time = 0.49, size = 68, normalized size = 1.21

$$\frac{\frac{6 \sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2 + 4\right)(\cos(dx+c)+1)}}{32d} - 5 \arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))^2,x, algorithm="maxima")

[Out] -1/32*(6*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d

Fricas [A]

time = 0.36, size = 59, normalized size = 1.05

$$\frac{5(3 \cos(dx + c) + 5) \arctan\left(\frac{5 \cos(dx+c)+3}{4 \sin(dx+c)}\right) + 12 \sin(dx + c)}{64(3d \cos(dx + c) + 5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))^2,x, algorithm="fricas")

[Out] -1/64*(5*(3*cos(d*x + c) + 5)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 12*sin(d*x + c))/(3*d*cos(d*x + c) + 5*d)

Sympy [C] Result contains complex when optimal does not.

time = 0.86, size = 187, normalized size = 3.34

$$\begin{cases} \frac{x}{(5+3 \cosh(2 \operatorname{atanh}(2)))^2} & \text{for } c = -dx - 2i \operatorname{atanh}(2) \vee c = -dx + 2i \operatorname{atanh}(2) \\ \frac{x}{(3 \cos(c)+5)^2} & \text{for } d = 0 \\ \frac{5 \left(\operatorname{atan}\left(\frac{\tan\left(\frac{\frac{c}{2} + \frac{dx}{2}}{2}\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor\right) \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{2}\right) + 20 \left(\operatorname{atan}\left(\frac{\tan\left(\frac{\frac{c}{2} + \frac{dx}{2}}{2}\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor\right) \right)}{32d \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{2}\right) + 128d} - \frac{6 \tan\left(\frac{\frac{c}{2} + \frac{dx}{2}}{2}\right)}{32d \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{2}\right) + 128d} \right)}{64d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))**2,x)

[Out] Piecewise((x/(5 + 3*cosh(2*atanh(2)))**2, Eq(c, -d*x - 2*I*atanh(2))) | Eq(c, -d*x + 2*I*atanh(2))), (x/(3*cos(c) + 5)**2, Eq(d, 0)), (5*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(32*d*tan(c/2 + d*x/2)**2 + 128*d) + 20*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(32*d*tan(c/2 + d*x/2)**2 + 128*d) - 6*tan(c/2 + d*x/2)/(32*d*tan(c/2 + d*x/2)**2 + 128*d), True))

Giac [A]

time = 0.43, size = 59, normalized size = 1.05

$$\frac{5 dx + 5 c - \frac{12 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4} - 10 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (5 \cdot dx + 5 \cdot c - 12 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 + 4) - 10 \cdot \arctan(\sin(dx + c) / (\cos(dx + c) + 3)) / d$

Mupad [B]

time = 0.33, size = 67, normalized size = 1.20

$$\frac{5 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{32d} - \frac{5 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{32d} - \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(c + d*x) + 5)^2,x)`

[Out] $\frac{5 \cdot \operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) / 2\right)}{32 \cdot d} - \frac{5 \cdot \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{32 \cdot d} - \frac{3 \cdot \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16 \cdot d \cdot \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 4\right)}$

$$3.20 \quad \int \frac{1}{(5+3 \cos(c+dx))^3} dx$$

Optimal. Leaf size=81

$$\frac{59x}{2048} - \frac{59 \operatorname{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d} - \frac{3 \sin(c+dx)}{32d(5+3 \cos(c+dx))^2} - \frac{45 \sin(c+dx)}{512d(5+3 \cos(c+dx))}$$

[Out] 59/2048*x-59/1024*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-3/32*sin(d*x+c)/d/(5+3*cos(d*x+c))^2-45/512*sin(d*x+c)/d/(5+3*cos(d*x+c))

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 2833, 12, 2736}

$$-\frac{59 \operatorname{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1024d} - \frac{45 \sin(c+dx)}{512d(3 \cos(c+dx)+5)} - \frac{3 \sin(c+dx)}{32d(3 \cos(c+dx)+5)^2} + \frac{59x}{2048}$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*Cos[c + d*x])^(-3),x]

[Out] (59*x)/2048 - (59*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(1024*d) - (3*Sin[c + d*x])/(32*d*(5 + 3*Cos[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(5 + 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2736

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(5 + 3 \cos(c + dx))^3} dx &= -\frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} - \frac{1}{32} \int \frac{-10 + 3 \cos(c + dx)}{(5 + 3 \cos(c + dx))^2} dx \\ &= -\frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))} + \frac{1}{512} \int \frac{59}{5 + 3 \cos(c + dx)} dx \\ &= -\frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))} + \frac{59}{512} \int \frac{1}{5 + 3 \cos(c + dx)} dx \\ &= \frac{59x}{2048} - \frac{59 \tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d} - \frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 56, normalized size = 0.69

$$-\frac{59 \operatorname{ArcTan}\left(2 \cot\left(\frac{1}{2}(c + dx)\right)\right) + \frac{3(182 \sin(c+dx) + 45 \sin(2(c+dx)))}{(5 + 3 \cos(c+dx))^2}}{1024d}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 3*Cos[c + d*x])^(-3), x]

[Out] -1/1024*(59*ArcTan[2*Cot[(c + d*x)/2]] + (3*(182*Sin[c + d*x] + 45*Sin[2*(c + d*x)]))/(5 + 3*Cos[c + d*x])^2)/d

Maple [A]

time = 0.08, size = 62, normalized size = 0.77

method	result	size
derivativedivides	$\frac{-\frac{69 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 51 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{128} - \frac{59 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{1024}}{4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)^2} + \frac{59 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{1024}$	62

default	$\frac{-\frac{69 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 51 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{128} - \frac{59 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{1024}}{4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)^2} + \frac{59 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{1024}}{d}$	62
risch	$-\frac{3i(59 e^{3i(dx+c)} + 295 e^{2i(dx+c)} + 241 e^{i(dx+c)} + 45)}{256d(3 e^{2i(dx+c)} + 10 e^{i(dx+c)} + 3)^2} - \frac{59i \ln(e^{i(dx+c)} + \frac{1}{3})}{2048d} + \frac{59i \ln(e^{i(dx+c)} + 3)}{2048d}$	105

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5+3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] `1/d*(1/4*(-69/128*tan(1/2*d*x+1/2*c)^3-51/32*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^2+59/1024*arctan(1/2*tan(1/2*d*x+1/2*c)))`

Maxima [A]

time = 0.55, size = 111, normalized size = 1.37

$$\frac{6 \left(\frac{68 \sin(dx+c)}{\cos(dx+c)+1} + \frac{23 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) - 59 \arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{\frac{8 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 16}}{1024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] `-1/1024*(6*(68*sin(d*x + c)/(cos(d*x + c) + 1) + 23*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 16) - 59*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d`

Fricas [A]

time = 0.39, size = 90, normalized size = 1.11

$$\frac{59(9 \cos(dx+c)^2 + 30 \cos(dx+c) + 25) \arctan\left(\frac{5 \cos(dx+c)+3}{4 \sin(dx+c)}\right) + 12(45 \cos(dx+c) + 91) \sin(dx+c)}{2048(9d \cos(dx+c)^2 + 30d \cos(dx+c) + 25d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] `-1/2048*(59*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 12*(45*cos(d*x + c) + 91)*sin(d*x + c))/(9*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 25*d)`

Sympy [C] Result contains complex when optimal does not.

time = 1.56, size = 359, normalized size = 4.43

$$\frac{\frac{1}{(5+3 \cosh(2 \operatorname{atanh}(2)))^2} - \frac{1}{(3 \cos(c)+5)^2}}{\frac{59 \left(\sin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right) + \frac{3 + \frac{59}{2} \frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}}{\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 47 \left(\sin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right) + \frac{3 + \frac{59}{2} \frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}}{\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 94 \left(\sin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right) + \frac{3 + \frac{59}{2} \frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}}{\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{1024d \tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) + 8192d \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 10384d} + \frac{47 \left(\sin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right) + \frac{3 + \frac{59}{2} \frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}}{\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 94 \left(\sin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right) + \frac{3 + \frac{59}{2} \frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}}{\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{1024d \tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) + 8192d \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 10384d} - \frac{138 \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{1024d \tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) + 8192d \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 10384d} - \frac{408 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1024d \tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) + 8192d \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 10384d}$$

for $c = -dx - 2i \operatorname{atanh}(2) \vee c = -dx + 2i \operatorname{atanh}(2)$
for $d = 0$
otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))**3,x)

[Out] Piecewise((x/(5 + 3*cosh(2*atanh(2)))**3, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c, -d*x + 2*I*atanh(2))), (x/(3*cos(c) + 5)**3, Eq(d, 0)), (59*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 472*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 944*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) - 138*tan(c/2 + d*x/2)**3/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) - 408*tan(c/2 + d*x/2)/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d), True))

Giac [A]

time = 0.51, size = 75, normalized size = 0.93

$$\frac{59 dx + 59 c - \frac{12 \left(23 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 68 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4 \right)^2} - 118 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/2048*(59*d*x + 59*c - 12*(23*tan(1/2*d*x + 1/2*c)^3 + 68*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 118*arctan(sin(d*x + c)/(cos(d*x + c) + 3))/d

Mupad [B]

time = 0.39, size = 96, normalized size = 1.19

$$\frac{59 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{1024 d} - \frac{59 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2} \right)}{1024 d} - \frac{\frac{69 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{512} + \frac{51 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{128}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 8 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 16 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cos(c + d*x) + 5)^3,x)

[Out] (59*atan(tan(c/2 + (d*x)/2)/2))/(1024*d) - (59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - ((51*tan(c/2 + (d*x)/2))/128 + (69*tan(c/2 + (d*x)/2)^3)/512)/(d*(8*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 16))

$$3.21 \quad \int \frac{1}{(5+3\cos(c+dx))^4} dx$$

Optimal. Leaf size=106

$$\frac{385x}{32768} - \frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{16384d} - \frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}$$

[Out] 385/32768*x-385/16384*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-1/16*sin(d*x+c)/d/(5+3*cos(d*x+c))^3-25/512*sin(d*x+c)/d/(5+3*cos(d*x+c))^2-311/8192*sin(d*x+c)/d/(5+3*cos(d*x+c))

Rubi [A]

time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 2833, 12, 2736}

$$-\frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{16384d} - \frac{311\sin(c+dx)}{8192d(3\cos(c+dx)+5)} - \frac{25\sin(c+dx)}{512d(3\cos(c+dx)+5)^2} - \frac{\sin(c+dx)}{16d(3\cos(c+dx)+5)^3} + \frac{385x}{32768}$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*Cos[c + d*x])^(-4), x]

[Out] (385*x)/32768 - (385*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(16384*d) - Sin[c + d*x]/(16*d*(5 + 3*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(5 + 3*Cos[c + d*x])^2) - (311*Sin[c + d*x])/(8192*d*(5 + 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2736

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5 + 3 \cos(c + dx))^4} dx &= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{1}{48} \int \frac{-15 + 6 \cos(c + dx)}{(5 + 3 \cos(c + dx))^3} dx \\
&= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} + \frac{\int \frac{186 - 75 \cos(c + dx)}{(5 + 3 \cos(c + dx))^2} dx}{1536} \\
&= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} - \frac{311 \sin(c + dx)}{8192d(5 + 3 \cos(c + dx))} \\
&= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} - \frac{311 \sin(c + dx)}{8192d(5 + 3 \cos(c + dx))} \\
&= \frac{385x}{32768} - \frac{385 \tan^{-1}\left(\frac{\sin(c + dx)}{3 + \cos(c + dx)}\right)}{16384d} - \frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2}
\end{aligned}$$

Mathematica [A]

time = 0.19, size = 66, normalized size = 0.62

$$-\frac{770 \operatorname{ArcTan}\left(2 \cot\left(\frac{1}{2}(c + dx)\right)\right) + \frac{9(4883 \sin(c + dx) + 2340 \sin(2(c + dx)) + 311 \sin(3(c + dx)))}{(5 + 3 \cos(c + dx))^3}}{32768d}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 3*Cos[c + d*x])^(-4), x]

[Out] -1/32768*(770*ArcTan[2*Cot[(c + d*x)/2]] + (9*(4883*Sin[c + d*x] + 2340*Sin[2*(c + d*x)] + 311*Sin[3*(c + d*x)]))/(5 + 3*Cos[c + d*x])^3)/d

Maple [A]

time = 0.10, size = 75, normalized size = 0.71

method	result
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derivativdivides	$\frac{-\frac{639(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{1024} - \frac{117(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{32} - \frac{369 \tan(\frac{dx}{2} + \frac{c}{2})}{64} + \frac{385 \arctan\left(\frac{\tan(\frac{dx}{2} + \frac{c}{2})}{2}\right)}{16384}}{8(\tan^2(\frac{dx}{2} + \frac{c}{2}) + 4)^3} + \frac{385 \arctan\left(\frac{\tan(\frac{dx}{2} + \frac{c}{2})}{2}\right)}{16384}}{d}$
default	$\frac{-\frac{639(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{1024} - \frac{117(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{32} - \frac{369 \tan(\frac{dx}{2} + \frac{c}{2})}{64} + \frac{385 \arctan\left(\frac{\tan(\frac{dx}{2} + \frac{c}{2})}{2}\right)}{16384}}{8(\tan^2(\frac{dx}{2} + \frac{c}{2}) + 4)^3} + \frac{385 \arctan\left(\frac{\tan(\frac{dx}{2} + \frac{c}{2})}{2}\right)}{16384}}{d}$
risch	$-\frac{i(10395 e^{5i(dx+c)} + 86625 e^{4i(dx+c)} + 239470 e^{3i(dx+c)} + 218466 e^{2i(dx+c)} + 73575 e^{i(dx+c)} + 8397)}{12288d(3 e^{2i(dx+c)} + 10 e^{i(dx+c)} + 3)^3} - \frac{385i \ln(e^{i(dx+c)} + 3)}{32768d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5+3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out] `1/d*(1/8*(-639/1024*tan(1/2*d*x+1/2*c)^5-117/32*tan(1/2*d*x+1/2*c)^3-369/64*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^3+385/16384*arctan(1/2*tan(1/2*d*x+1/2*c))`

Maxima [A]

time = 0.49, size = 151, normalized size = 1.42

$$\frac{18 \left(\frac{656 \sin(dx+c)}{\cos(dx+c)+1} + \frac{416 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{71 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) - 385 \arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{\frac{48 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{12 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 64} 16384 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*cos(d*x+c))^4,x, algorithm="maxima")`

[Out] `-1/16384*(18*(656*sin(d*x + c)/(cos(d*x + c) + 1) + 416*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 71*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 385*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d`

Fricas [A]

time = 0.41, size = 121, normalized size = 1.14

$$\frac{385 (27 \cos(dx+c)^3 + 135 \cos(dx+c)^2 + 225 \cos(dx+c) + 125) \arctan\left(\frac{5 \cos(dx+c)+3}{4 \sin(dx+c)}\right) + 36 (311 \cos(dx+c)^2 + 1170 \cos(dx+c) + 1143) \sin(dx+c)}{32768 (27 d \cos(dx+c)^3 + 135 d \cos(dx+c)^2 + 225 d \cos(dx+c) + 125 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] `-1/32768*(385*(27*cos(d*x + c)^3 + 135*cos(d*x + c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 36*(311*cos(d*x + c)^2`

+ 1170*cos(d*x + c) + 1143)*sin(d*x + c))/(27*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)

Sympy [C] Result contains complex when optimal does not.

time = 3.26, size = 592, normalized size = 5.58



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))**4,x)

[Out] Piecewise((x/(5 + 3*cosh(2*atanh(2)))**4, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c, -d*x + 2*I*atanh(2))), (x/(3*cos(c) + 5)**4, Eq(d, 0)), (385*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 4620*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 18480*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 24640*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 1278*tan(c/2 + d*x/2)**5/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 7488*tan(c/2 + d*x/2)**3/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 11808*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d), True))

Giac [A]

time = 0.43, size = 88, normalized size = 0.83

$$\frac{385 dx + 385 c - \frac{36 \left(71 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 416 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 656 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4 \right)^3} - 770 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{32768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/32768*(385*d*x + 385*c - 36*(71*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x + 1/2*c)^3 + 656*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^3 - 770*arctan(sin(d*x + c)/(cos(d*x + c) + 3))/d

Mupad [B]

time = 0.52, size = 96, normalized size = 0.91

$$\frac{385 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{16384 d} - \frac{385 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{16384 d} - \frac{\frac{639 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8192} + \frac{117 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{256} + \frac{369 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{512}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(c + d*x) + 5)^4,x)`

[Out] `(385*atan(tan(c/2 + (d*x)/2)/2))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2) - (d*x)/2))/(16384*d) - ((369*tan(c/2 + (d*x)/2))/512 + (117*tan(c/2 + (d*x)/2)^3)/256 + (639*tan(c/2 + (d*x)/2)^5)/8192)/(d*(tan(c/2 + (d*x)/2)^2 + 4)^3)`

$$3.22 \quad \int \frac{1}{5-3 \cos(c+dx)} dx$$

Optimal. Leaf size=33

$$\frac{x}{4} + \frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

[Out] 1/4*x+1/2*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2736}

$$\frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d} + \frac{x}{4}$$

Antiderivative was successfully verified.

[In] Int[(5 - 3*Cos[c + d*x])^(-1),x]

[Out] x/4 + ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])]/(2*d)

Rule 2736

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rubi steps

$$\int \frac{1}{5-3 \cos(c+dx)} dx = \frac{x}{4} + \frac{\tan^{-1}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 0.61

$$\frac{\text{ArcTan}\left(2 \tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 3*Cos[c + d*x])^(-1),x]

[Out] ArcTan[2*Tan[(c + d*x)/2]]/(2*d)

Maple [A]

time = 0.05, size = 18, normalized size = 0.55

method	result	size
derivativedivides	$\frac{\arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d}$	18
default	$\frac{\arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d}$	18
risch	$\frac{i \ln(e^{i(dx+c)} - 3)}{4d} - \frac{i \ln(e^{i(dx+c)} - \frac{1}{3})}{4d}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5-3*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/2/d*arctan(2*tan(1/2*d*x+1/2*c))

Maxima [A]

time = 0.50, size = 24, normalized size = 0.73

$$\frac{\arctan\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/2*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1))/d

Fricas [A]

time = 0.38, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{5 \cos(dx+c)-3}{4 \sin(dx+c)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c)),x, algorithm="fricas")

[Out] -1/4*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c))/d

Sympy [A]

time = 0.34, size = 41, normalized size = 1.24

$$\begin{cases} \frac{\operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor}{2d} & \text{for } d \neq 0 \\ \frac{x}{5-3 \cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c)),x)

[Out] Piecewise(((atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(2*d), Ne(d, 0)), (x/(5 - 3*cos(c)), True))

Giac [A]

time = 0.43, size = 30, normalized size = 0.91

$$\frac{dx + c - 2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c)),x, algorithm="giac")

[Out] 1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d

Mupad [B]

time = 0.29, size = 38, normalized size = 1.15

$$\frac{\operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{2d} - \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(3*cos(c + d*x) - 5),x)

[Out] atan(2*tan(c/2 + (d*x)/2))/(2*d) - (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d)

3.23 $\int \frac{1}{(5-3\cos(c+dx))^2} dx$

Optimal. Leaf size=58

$$\frac{5x}{64} + \frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}$$

[Out] 5/64*x+5/32*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+3/16*sin(d*x+c)/d/(5-3*cos(d*x+c))

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2743, 12, 2736}

$$\frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} + \frac{5x}{64}$$

Antiderivative was successfully verified.

[In] Int[(5 - 3*Cos[c + d*x])^(-2), x]

[Out] (5*x)/64 + (5*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(32*d) + (3*Sin[c + d*x])/(16*d*(5 - 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2736

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(5-3\cos(c+dx))^2} dx &= \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} - \frac{1}{16} \int \frac{5}{5-3\cos(c+dx)} dx \\ &= \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} + \frac{5}{16} \int \frac{1}{5-3\cos(c+dx)} dx \\ &= \frac{5x}{64} + \frac{5 \tan^{-1}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 43, normalized size = 0.74

$$\frac{5\text{ArcTan}\left(2 \tan\left(\frac{1}{2}(c+dx)\right)\right) - \frac{6 \sin(c+dx)}{-5+3 \cos(c+dx)}}{32d}$$

Antiderivative was successfully verified.

`[In] Integrate[(5 - 3*Cos[c + d*x])^(-2), x]``[Out] (5*ArcTan[2*Tan[(c + d*x)/2]] - (6*Sin[c + d*x])/(-5 + 3*Cos[c + d*x]))/(32*d)`**Maple [A]**

time = 0.07, size = 46, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{64\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{1}{4}\right)} + \frac{5 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{32}}{d}$	46
default	$\frac{\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{64\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{1}{4}\right)} + \frac{5 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{32}}{d}$	46
risch	$\frac{i(5e^{i(dx+c)}-3)}{8d(3e^{2i(dx+c)}-10e^{i(dx+c)}+3)} + \frac{5i \ln(e^{i(dx+c)}-3)}{64d} - \frac{5i \ln(e^{i(dx+c)}-\frac{1}{3})}{64d}$	83

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(5-3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)``[Out] 1/d*(3/64*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1/4)+5/32*arctan(2*tan(1/2*d*x+1/2*c)))`**Maxima [A]**

time = 0.50, size = 69, normalized size = 1.19

$$\frac{\frac{6 \sin(dx+c)}{\left(\frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1\right)(\cos(dx+c)+1)}}{32d} + 5 \arctan\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/32*(6*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)) + 5*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d

Fricas [A]

time = 0.37, size = 59, normalized size = 1.02

$$\frac{5(3 \cos(dx + c) - 5) \arctan\left(\frac{5 \cos(dx+c)-3}{4 \sin(dx+c)}\right) + 12 \sin(dx + c)}{64(3d \cos(dx + c) - 5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c))^2,x, algorithm="fricas")

[Out] -1/64*(5*(3*cos(d*x + c) - 5)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c)) + 12*sin(d*x + c))/(3*d*cos(d*x + c) - 5*d)

Sympy [C] Result contains complex when optimal does not.

time = 0.71, size = 192, normalized size = 3.31

$$\begin{cases} \frac{x}{(5-3 \cosh(2 \operatorname{atanh}(\frac{1}{2})))^2} & \text{for } c = -dx - 2i \operatorname{atanh}(\frac{1}{2}) \vee c = -dx + 2i \operatorname{atanh}(\frac{1}{2}) \\ \frac{x}{(5-3 \cos(c))^2} & \text{for } d = 0 \\ \frac{20 \left(\operatorname{atan}\left(2 \tan\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right) + \pi \left| \frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}} - \frac{\pi}{2} \right| \right) \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right)}{128d \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right) + 32d} + \frac{5 \left(\operatorname{atan}\left(2 \tan\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right) + \pi \left| \frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}} - \frac{\pi}{2} \right| \right) \right)}{128d \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right) + 32d} + \frac{6 \tan\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right)}{128d \tan^2\left(\frac{\frac{c}{2} + \frac{dx}{2}}{\frac{5}{2} + \frac{dx}{2}}\right) + 32d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c))*2,x)

[Out] Piecewise((x/(5 - 3*cosh(2*atanh(1/2))))**2, Eq(c, -d*x - 2*I*atanh(1/2)) | Eq(c, -d*x + 2*I*atanh(1/2))), (x/(5 - 3*cos(c))**2, Eq(d, 0)), (20*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 5*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 6*tan(c/2 + d*x/2)/(128*d*tan(c/2 + d*x/2)**2 + 32*d), True))

Giac [A]

time = 0.44, size = 61, normalized size = 1.05

$$\frac{5 dx + 5 c + \frac{12 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1} - 10 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (5 \cdot d \cdot x + 5 \cdot c + 12 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) / (4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 1) - 10 \cdot \arctan(\sin(d \cdot x + c) / (\cos(d \cdot x + c) - 3)) / d$

Mupad [B]

time = 0.31, size = 67, normalized size = 1.16

$$\frac{5 \operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{32 d} - \frac{5 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{32 d} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{64 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(1/(3 \cdot \cos(c + d \cdot x) - 5)^2, x)$

[Out] $(5 \cdot \operatorname{atan}(2 \cdot \tan(c/2 + (d \cdot x)/2))) / (32 \cdot d) - (5 \cdot (\operatorname{atan}(\tan(c/2 + (d \cdot x)/2)) - (d \cdot x)/2)) / (32 \cdot d) + (3 \cdot \tan(c/2 + (d \cdot x)/2)) / (64 \cdot d \cdot (\tan(c/2 + (d \cdot x)/2)^2 + 1/4))$

3.24 $\int \frac{1}{(5-3\cos(c+dx))^3} dx$

Optimal. Leaf size=83

$$\frac{59x}{2048} + \frac{59\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} + \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}$$

[Out] 59/2048*x+59/1024*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+3/32*sin(d*x+c)/d/(5-3*cos(d*x+c))^2+45/512*sin(d*x+c)/d/(5-3*cos(d*x+c))

Rubi [A]

time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {2743, 2833, 12, 2736}

$$\frac{59\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} + \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} + \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} + \frac{59x}{2048}$$

Antiderivative was successfully verified.

[In] Int[(5 - 3*Cos[c + d*x])^(-3),x]

[Out] (59*x)/2048 + (59*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(1024*d) + (3*Sin[c + d*x])/(32*d*(5 - 3*Cos[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(5 - 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2736

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(5 - 3 \cos(c + dx))^3} dx &= \frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} - \frac{1}{32} \int \frac{-10 - 3 \cos(c + dx)}{(5 - 3 \cos(c + dx))^2} dx \\ &= \frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))} + \frac{1}{512} \int \frac{59}{5 - 3 \cos(c + dx)} dx \\ &= \frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))} + \frac{59}{512} \int \frac{1}{5 - 3 \cos(c + dx)} dx \\ &= \frac{59x}{2048} + \frac{59 \tan^{-1}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} + \frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))} \end{aligned}$$

Mathematica [A]

time = 0.10, size = 65, normalized size = 0.78

$$\frac{59 \operatorname{ArcTan}\left(2 \tan\left(\frac{1}{2}(c + dx)\right)\right) (5 - 3 \cos(c + dx))^2 + 546 \sin(c + dx) - 135 \sin(2(c + dx))}{1024d(5 - 3 \cos(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 3*Cos[c + d*x])^(-3), x]

[Out] (59*ArcTan[2*Tan[(c + d*x)/2]]*(5 - 3*Cos[c + d*x])^2 + 546*Sin[c + d*x] - 135*Sin[2*(c + d*x)])/(1024*d*(5 - 3*Cos[c + d*x])^2)

Maple [A]

time = 0.08, size = 64, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\frac{51 \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{128} + \frac{69 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{512} + \frac{59 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{1024}}{\left(4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)^2} + \frac{59 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{1024}}$	64
default	$\frac{\frac{51 \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{128} + \frac{69 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{512} + \frac{59 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{1024}}{\left(4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)^2} + \frac{59 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{1024}}$	64

risch	$\frac{3i(59e^{3i(dx+c)} - 295e^{2i(dx+c)} + 241e^{i(dx+c)} - 45)}{256d(3e^{2i(dx+c)} - 10e^{i(dx+c)} + 3)^2} + \frac{59i \ln(e^{i(dx+c)} - 3)}{2048d} - \frac{59i \ln(e^{i(dx+c)} - \frac{1}{3})}{2048d}$	105
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5-3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] $1/d*(4*(51/512*\tan(1/2*d*x+1/2*c))^3+69/2048*\tan(1/2*d*x+1/2*c))/(4*\tan(1/2*d*x+1/2*c)^2+1)^2+59/1024*\arctan(2*\tan(1/2*d*x+1/2*c))$

Maxima [A]

time = 0.51, size = 112, normalized size = 1.35

$$\frac{6 \left(\frac{23 \sin(dx+c)}{\cos(dx+c)+1} + \frac{68 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{\frac{8 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{16 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1} + 59 \arctan \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} \right)$$

1024 d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5-3*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $1/1024*(6*(23*\sin(dx+c)/(\cos(dx+c)+1)+68*\sin(dx+c)^3/(\cos(dx+c)+1)^3)/(8*\sin(dx+c)^2/(\cos(dx+c)+1)^2+16*\sin(dx+c)^4/(\cos(dx+c)+1)^4+1)+59*\arctan(2*\sin(dx+c)/(\cos(dx+c)+1)))/d$

Fricas [A]

time = 0.36, size = 90, normalized size = 1.08

$$\frac{59(9 \cos(dx+c)^2 - 30 \cos(dx+c) + 25) \arctan\left(\frac{5 \cos(dx+c) - 3}{4 \sin(dx+c)}\right) + 12(45 \cos(dx+c) - 91) \sin(dx+c)}{2048(9d \cos(dx+c)^2 - 30d \cos(dx+c) + 25d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5-3*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $-1/2048*(59*(9*\cos(dx+c)^2 - 30*\cos(dx+c) + 25)*\arctan(1/4*(5*\cos(dx+c) - 3)/\sin(dx+c)) + 12*(45*\cos(dx+c) - 91)*\sin(dx+c))/(9*d*\cos(dx+c)^2 - 30*d*\cos(dx+c) + 25*d)$

Sympy [C] Result contains complex when optimal does not.

time = 1.44, size = 364, normalized size = 4.39

$$\left(\frac{(5-3 \cosh(2i \operatorname{atanh}(\frac{1}{2})))^3}{(5-3 \cos(c))^3} \right)^{\frac{1}{3}}$$

for $c = -dx - 2i \operatorname{atanh}(\frac{1}{2}) \vee c = -dx + 2i \operatorname{atanh}(\frac{1}{2})$
for $d = 0$

$$\left(\frac{94i \left(\operatorname{atan}\left(2 \tan\left(\frac{1}{2} + \frac{1}{2}i\right)\right) + i \frac{1+2i \tan\left(\frac{1}{2} + \frac{1}{2}i\right)}{1+2i \tan\left(\frac{1}{2} + \frac{1}{2}i\right)} \right) \tan^2\left(\frac{1}{2} + \frac{1}{2}i\right) + 472 \left(\operatorname{atan}\left(2 \tan\left(\frac{1}{2} + \frac{1}{2}i\right)\right) + i \frac{1+2i \tan\left(\frac{1}{2} + \frac{1}{2}i\right)}{1+2i \tan\left(\frac{1}{2} + \frac{1}{2}i\right)} \right) \tan^2\left(\frac{1}{2} + \frac{1}{2}i\right) + \frac{92 \left(\operatorname{atan}\left(2 \tan\left(\frac{1}{2} + \frac{1}{2}i\right)\right) + i \frac{1+2i \tan\left(\frac{1}{2} + \frac{1}{2}i\right)}{1+2i \tan\left(\frac{1}{2} + \frac{1}{2}i\right)} \right) \tan^2\left(\frac{1}{2} + \frac{1}{2}i\right)}{16384 \tan^4\left(\frac{1}{2} + \frac{1}{2}i\right) + 8192i \tan^3\left(\frac{1}{2} + \frac{1}{2}i\right) + 1024d} + \frac{408 \tan^2\left(\frac{1}{2} + \frac{1}{2}i\right)}{16384 \tan^4\left(\frac{1}{2} + \frac{1}{2}i\right) + 8192i \tan^3\left(\frac{1}{2} + \frac{1}{2}i\right) + 1024d} + \frac{138 \tan\left(\frac{1}{2} + \frac{1}{2}i\right)}{16384 \tan^4\left(\frac{1}{2} + \frac{1}{2}i\right) + 8192i \tan^3\left(\frac{1}{2} + \frac{1}{2}i\right) + 1024d} \right)^{\frac{1}{3}}$$

otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5-3*cos(d*x+c))**3,x)`

```
[Out] Piecewise((x/(5 - 3*cosh(2*atanh(1/2)))**3, Eq(c, -d*x - 2*I*atanh(1/2)) |
Eq(c, -d*x + 2*I*atanh(1/2))), (x/(5 - 3*cos(c))**3, Eq(d, 0)), (944*(atan(
2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**
4/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) + 472
*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d
*x/2)**2/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d
) + 59*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1638
4*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) + 408*tan(c/
2 + d*x/2)**3/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1
024*d) + 138*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2
+ d*x/2)**2 + 1024*d), True))
```

Giac [A]

time = 0.45, size = 77, normalized size = 0.93

$$\frac{59 dx + 59 c + \frac{12 \left(68 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 23 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^2} - 118 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-3*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(59*d*x + 59*c + 12*(68*tan(1/2*d*x + 1/2*c)^3 + 23*tan(1/2*d*x + 1/
2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 118*arctan(sin(d*x + c)/(cos(d*x +
c) - 3)))/d
```

Mupad [B]

time = 0.37, size = 95, normalized size = 1.14

$$\frac{59 \operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{1024 d} - \frac{59 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{1024 d} + \frac{\frac{51 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{2048} + \frac{69 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8192}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{2} + \frac{1}{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*cos(c + d*x) - 5)^3,x)
```

```
[Out] (59*atan(2*tan(c/2 + (d*x)/2)))/(1024*d) - (59*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2))/(1024*d) + ((69*tan(c/2 + (d*x)/2))/8192 + (51*atan(c/2 + (d*x)/2)
^3)/2048)/(d*(tan(c/2 + (d*x)/2)^2/2 + tan(c/2 + (d*x)/2)^4 + 1/16))
```

$$3.25 \quad \int \frac{1}{(5-3\cos(c+dx))^4} dx$$

Optimal. Leaf size=108

$$\frac{385x}{32768} + \frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}$$

[Out] 385/32768*x+385/16384*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+1/16*sin(d*x+c)/d/(5-3*cos(d*x+c))^3+25/512*sin(d*x+c)/d/(5-3*cos(d*x+c))^2+311/8192*sin(d*x+c)/d/(5-3*cos(d*x+c))

Rubi [A]

time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 2833, 12, 2736}

$$\frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{385x}{32768}$$

Antiderivative was successfully verified.

[In] Int[(5 - 3*Cos[c + d*x])^(-4), x]

[Out] (385*x)/32768 + (385*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(16384*d) + Sin[c + d*x]/(16*d*(5 - 3*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(5 - 3*Cos[c + d*x])^2) + (311*Sin[c + d*x])/(8192*d*(5 - 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2736

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5 - 3 \cos(c + dx))^4} dx &= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} - \frac{1}{48} \int \frac{-15 - 6 \cos(c + dx)}{(5 - 3 \cos(c + dx))^3} dx \\
&= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} + \frac{\int \frac{186 + 75 \cos(c + dx)}{(5 - 3 \cos(c + dx))^2} dx}{1536} \\
&= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} + \frac{311 \sin(c + dx)}{8192d(5 - 3 \cos(c + dx))} \\
&= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} + \frac{311 \sin(c + dx)}{8192d(5 - 3 \cos(c + dx))} \\
&= \frac{385x}{32768} + \frac{385 \tan^{-1}\left(\frac{\sin(c + dx)}{3 - \cos(c + dx)}\right)}{16384d} + \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2}
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 66, normalized size = 0.61

$$\frac{770 \operatorname{ArcTan}\left(2 \tan\left(\frac{1}{2}(c + dx)\right)\right) - \frac{9(4883 \sin(c + dx) - 2340 \sin(2(c + dx)) + 311 \sin(3(c + dx)))}{(-5 + 3 \cos(c + dx))^3}}{32768d}$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 3*Cos[c + d*x])^(-4), x]

[Out] (770*ArcTan[2*Tan[(c + d*x)/2]] - (9*(4883*Sin[c + d*x] - 2340*Sin[2*(c + d*x)] + 311*Sin[3*(c + d*x)]))/(-5 + 3*Cos[c + d*x])^3)/(32768*d)

Maple [A]

time = 0.10, size = 77, normalized size = 0.71

method	result
--------	--------

derivativedivides	$\frac{\frac{369 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512} + \frac{117 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{256} + \frac{639 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8192} + \frac{385 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16384}}{\left(4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)^3} \cdot d$
default	$\frac{\frac{369 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512} + \frac{117 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{256} + \frac{639 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8192} + \frac{385 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16384}}{\left(4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)^3} \cdot d$
risch	$\frac{i(10395 e^{5i(dx+c)} - 86625 e^{4i(dx+c)} + 239470 e^{3i(dx+c)} - 218466 e^{2i(dx+c)} + 73575 e^{i(dx+c)} - 8397)}{12288d(3e^{2i(dx+c)} - 10e^{i(dx+c)} + 3)^3} + \frac{385i \ln(e^{i(dx+c)} - 3)}{32768d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5-3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{d} \cdot \left(8 \cdot \left(\frac{369}{4096} \tan^5\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{117}{2048} \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{639}{65536} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right) \cdot \left(4 \tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1 \right)^3 + \frac{385}{16384} \arctan\left(2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) \right)$

Maxima [A]

time = 0.49, size = 152, normalized size = 1.41

$$\frac{18 \left(\frac{71 \sin(dx+c)}{\cos(dx+c)+1} + \frac{416 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{656 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{\frac{12 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{48 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{64 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1} + 385 \arctan\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1}\right)$$

$16384 d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5-3*cos(d*x+c))^4,x, algorithm="maxima")`

[Out] $\frac{1}{16384} \cdot \left(18 \cdot \left(\frac{71 \sin(dx+c)}{\cos(dx+c)+1} + \frac{416 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{656 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) \cdot \left(\frac{12 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{48 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{64 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right) + 385 \arctan\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1}\right) \right) / d$

Fricas [A]

time = 0.37, size = 121, normalized size = 1.12

$$\frac{385 (27 \cos(dx+c)^3 - 135 \cos(dx+c)^2 + 225 \cos(dx+c) - 125) \arctan\left(\frac{5 \cos(dx+c) - 3}{4 \sin(dx+c)}\right) + 36 (311 \cos(dx+c)^2 - 1170 \cos(dx+c) + 1143) \sin(dx+c)}{32768 (27 d \cos(dx+c)^3 - 135 d \cos(dx+c)^2 + 225 d \cos(dx+c) - 125 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5-3*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] $-1/32768 \cdot \left(385 \cdot \left(27 \cos^3(dx+c) - 135 \cos^2(dx+c) + 225 \cos(dx+c) - 125 \right) \cdot \arctan\left(\frac{5 \cos(dx+c) - 3}{4 \sin(dx+c)}\right) + 36 \cdot \left(311 \cos^2(dx+c) - 1170 \cos(dx+c) + 1143 \right) \cdot \sin(dx+c) \right) / \left(27 d \cos^3(dx+c) - 135 d \cos^2(dx+c) + 225 d \cos(dx+c) - 125 d \right)$

Sympy [C] Result contains complex when optimal does not.

time = 3.17, size = 597, normalized size = 5.53



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c))**4,x)

[Out] Piecewise((x/(5 - 3*cosh(2*atanh(1/2)))**4, Eq(c, -d*x - 2*I*atanh(1/2)) | Eq(c, -d*x + 2*I*atanh(1/2))), (x/(5 - 3*cos(c))**4, Eq(d, 0)), (24640*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 18480*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 4620*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 385*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 11808*tan(c/2 + d*x/2)**5/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 7488*tan(c/2 + d*x/2)**3/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 1278*tan(c/2 + d*x/2)/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d), True))

Giac [A]

time = 0.49, size = 90, normalized size = 0.83

$$\frac{385 dx + 385 c + \frac{36 \left(656 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 416 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 71 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\left(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right)^3} - 770 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{32768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-3*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/32768*(385*d*x + 385*c + 36*(656*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x + 1/2*c)^3 + 71*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^3 - 770*arctan(sin(d*x + c)/(cos(d*x + c) - 3))/d

Mupad [B]

time = 0.49, size = 97, normalized size = 0.90

$$\frac{385 \operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{16384 d} - \frac{385 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{16384 d} + \frac{\frac{369 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{512} + \frac{117 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{256} + \frac{639 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8192}}{d \left(4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(c + d*x) - 5)^4,x)
```

```
[Out] (385*atan(2*tan(c/2 + (d*x)/2)))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(16384*d) + ((639*tan(c/2 + (d*x)/2))/8192 + (117*tan(c/2 + (d*x)/2)^3)/256 + (369*tan(c/2 + (d*x)/2)^5)/512)/(d*(4*tan(c/2 + (d*x)/2)^2 + 1)^3)
```


$$3.26 \quad \int \frac{1}{-5+3 \cos(c+dx)} dx$$

Optimal. Leaf size=33

$$-\frac{x}{4} - \frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

[Out] -1/4*x-1/2*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2737}

$$-\frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d} - \frac{x}{4}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 3*Cos[c + d*x])^(-1), x]

[Out] -1/4*x - ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])]/(2*d)

Rule 2737

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] & NegQ[a]

Rubi steps

$$\int \frac{1}{-5+3 \cos(c+dx)} dx = -\frac{x}{4} - \frac{\tan^{-1}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 0.61

$$-\frac{\text{ArcTan}\left(2 \tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 3*Cos[c + d*x])^(-1), x]

[Out] $-1/2*\text{ArcTan}[2*\text{Tan}[(c + d*x)/2]]/d$

Maple [A]

time = 0.05, size = 18, normalized size = 0.55

method	result	size
derivativedivides	$-\frac{\arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d}$	18
default	$-\frac{\arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d}$	18
risch	$-\frac{i \ln(e^{i(dx+c)}-3)}{4d} + \frac{i \ln(e^{i(dx+c)}-\frac{1}{3})}{4d}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-5+3*cos(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] $-1/2/d*\arctan(2*\tan(1/2*d*x+1/2*c))$

Maxima [A]

time = 0.50, size = 24, normalized size = 0.73

$$-\frac{\arctan\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+3*cos(d*x+c)),x, algorithm="maxima")`

[Out] $-1/2*\arctan(2*\sin(d*x + c)/(\cos(d*x + c) + 1))/d$

Fricas [A]

time = 0.36, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{5 \cos(dx+c)-3}{4 \sin(dx+c)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+3*cos(d*x+c)),x, algorithm="fricas")`

[Out] $1/4*\arctan(1/4*(5*\cos(d*x + c) - 3)/\sin(d*x + c))/d$

Sympy [A]

time = 0.33, size = 42, normalized size = 1.27

$$\begin{cases} -\frac{\text{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor}{2d} & \text{for } d \neq 0 \\ \frac{x}{3 \cos(c) - 5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c)),x)

[Out] Piecewise((-atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(2*d), Ne(d, 0)), (x/(3*cos(c) - 5), True))

Giac [A]

time = 0.42, size = 30, normalized size = 0.91

$$-\frac{dx + c - 2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c)),x, algorithm="giac")

[Out] -1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d

Mupad [B]

time = 0.25, size = 38, normalized size = 1.15

$$\frac{\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}}{2d} - \frac{\operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cos(c + d*x) - 5),x)

[Out] (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d) - atan(2*tan(c/2 + (d*x)/2))/(2*d)

$$3.27 \quad \int \frac{1}{(-5+3\cos(c+dx))^2} dx$$

Optimal. Leaf size=58

$$\frac{5x}{64} + \frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}$$

[Out] 5/64*x+5/32*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+3/16*sin(d*x+c)/d/(5-3*cos(d*x+c))

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2743, 12, 2737}

$$\frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} + \frac{5x}{64}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 3*Cos[c + d*x])^(-2), x]

[Out] (5*x)/64 + (5*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(32*d) + (3*Sin[c + d*x])/(16*d*(5 - 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2737

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] & NegQ[a]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 + 3 \cos(c + dx))^2} dx &= \frac{3 \sin(c + dx)}{16d(5 - 3 \cos(c + dx))} - \frac{1}{16} \int \frac{5}{-5 + 3 \cos(c + dx)} dx \\ &= \frac{3 \sin(c + dx)}{16d(5 - 3 \cos(c + dx))} - \frac{5}{16} \int \frac{1}{-5 + 3 \cos(c + dx)} dx \\ &= \frac{5x}{64} + \frac{5 \tan^{-1}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3 \sin(c + dx)}{16d(5 - 3 \cos(c + dx))} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 0.74

$$\frac{5 \operatorname{ArcTan}\left(2 \tan\left(\frac{1}{2}(c + dx)\right)\right) - \frac{6 \sin(c+dx)}{-5+3 \cos(c+dx)}}{32d}$$

Antiderivative was successfully verified.

`[In] Integrate[(-5 + 3*Cos[c + d*x])^(-2), x]``[Out] (5*ArcTan[2*Tan[(c + d*x)/2]] - (6*Sin[c + d*x])/(-5 + 3*Cos[c + d*x]))/(32*d)`**Maple [A]**

time = 0.06, size = 46, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{64 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{1}{4}\right)} + \frac{5 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{32}}{d}$	46
default	$\frac{\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{64 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{1}{4}\right)} + \frac{5 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{32}}{d}$	46
risch	$\frac{i(5 e^{i(dx+c)} - 3)}{8d(3 e^{2i(dx+c)} - 10 e^{i(dx+c)} + 3)} + \frac{5i \ln(e^{i(dx+c)} - 3)}{64d} - \frac{5i \ln(e^{i(dx+c)} - \frac{1}{3})}{64d}$	83

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-5+3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)``[Out] 1/d*(3/64*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1/4)+5/32*arctan(2*tan(1/2*d*x+1/2*c)))`**Maxima [A]**

time = 0.48, size = 69, normalized size = 1.19

$$\frac{\frac{6 \sin(dx+c)}{\left(\frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1\right)(\cos(dx+c)+1)} + 5 \arctan\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1}\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/32*(6*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)) + 5*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d

Fricas [A]

time = 0.36, size = 59, normalized size = 1.02

$$\frac{5(3 \cos(dx + c) - 5) \arctan\left(\frac{5 \cos(dx+c)-3}{4 \sin(dx+c)}\right) + 12 \sin(dx + c)}{64(3d \cos(dx + c) - 5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))^2,x, algorithm="fricas")

[Out] -1/64*(5*(3*cos(d*x + c) - 5)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c)) + 12*sin(d*x + c))/(3*d*cos(d*x + c) - 5*d)

Sympy [C] Result contains complex when optimal does not.

time = 0.73, size = 192, normalized size = 3.31

$$\begin{cases} \frac{x}{(-5+3 \cosh(2 \operatorname{atanh}(\frac{1}{2})))^2} & \text{for } c = -dx - 2i \operatorname{atanh}(\frac{1}{2}) \vee c = -dx + 2i \operatorname{atanh}(\frac{1}{2}) \\ \frac{x}{(3 \cos(c)-5)^2} & \text{for } d = 0 \\ \frac{20 \left(\operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right) \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{128d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 32d} + \frac{5 \left(\operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{128d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 32d} + \frac{6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{128d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 32d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))^2,x)

[Out] Piecewise((x/(-5 + 3*cosh(2*atanh(1/2))))**2, Eq(c, -d*x - 2*I*atanh(1/2)) | Eq(c, -d*x + 2*I*atanh(1/2))), (x/(3*cos(c) - 5)**2, Eq(d, 0)), (20*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 5*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 6*tan(c/2 + d*x/2)/(128*d*tan(c/2 + d*x/2)**2 + 32*d), True))

Giac [A]

time = 0.47, size = 61, normalized size = 1.05

$$\frac{5 dx + 5 c + \frac{12 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1} - 10 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (5 \cdot d \cdot x + 5 \cdot c + 12 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) / (4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 1) - 10 \cdot \arctan(\sin(d \cdot x + c) / (\cos(d \cdot x + c) - 3)) / d$

Mupad [B]

time = 0.00, size = 67, normalized size = 1.16

$$\frac{5 \operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{32 d} - \frac{5 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{32 d} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{64 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(c + d*x) - 5)^2,x)`

[Out] $\frac{5 \cdot \operatorname{atan}(2 \cdot \tan(c/2 + (d \cdot x)/2))}{(32 \cdot d)} - \frac{5 \cdot (\operatorname{atan}(\tan(c/2 + (d \cdot x)/2)) - (d \cdot x)/2)}{(32 \cdot d)} + \frac{3 \cdot \tan(c/2 + (d \cdot x)/2)}{(64 \cdot d \cdot (\tan(c/2 + (d \cdot x)/2)^2 + 1/4))}$

$$3.28 \quad \int \frac{1}{(-5+3\cos(c+dx))^3} dx$$

Optimal. Leaf size=83

$$-\frac{59x}{2048} - \frac{59\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} - \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}$$

[Out] -59/2048*x-59/1024*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d-3/32*sin(d*x+c)/d/(5-3*cos(d*x+c))^2-45/512*sin(d*x+c)/d/(5-3*cos(d*x+c))

Rubi [A]

time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {2743, 2833, 12, 2737}

$$-\frac{59\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} - \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{59x}{2048}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 3*Cos[c + d*x])^(-3), x]

[Out] (-59*x)/2048 - (59*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(1024*d) - (3*Sin[c + d*x])/(32*d*(5 - 3*Cos[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(5 - 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2737

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && NegQ[a]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 + 3 \cos(c + dx))^3} dx &= -\frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} - \frac{1}{32} \int \frac{10 + 3 \cos(c + dx)}{(-5 + 3 \cos(c + dx))^2} dx \\ &= -\frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))} + \frac{1}{512} \int \frac{59}{-5 + 3 \cos(c + dx)} dx \\ &= -\frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))} + \frac{59}{512} \int \frac{1}{-5 + 3 \cos(c + dx)} dx \\ &= -\frac{59x}{2048} - \frac{59 \tan^{-1}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} - \frac{3 \sin(c + dx)}{32d(5 - 3 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 65, normalized size = 0.78

$$\frac{-59 \operatorname{ArcTan}\left(2 \tan\left(\frac{1}{2}(c + dx)\right)\right) (5 - 3 \cos(c + dx))^2 - 546 \sin(c + dx) + 135 \sin(2(c + dx))}{1024d(5 - 3 \cos(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 3*Cos[c + d*x])^(-3), x]

[Out] (-59*ArcTan[2*Tan[(c + d*x)/2]]*(5 - 3*Cos[c + d*x])^2 - 546*Sin[c + d*x] + 135*Sin[2*(c + d*x)])/(1024*d*(5 - 3*Cos[c + d*x])^2)

Maple [A]

time = 0.08, size = 64, normalized size = 0.77

method	result	size
derivativedivides	$\frac{4 \left(\frac{51 \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{512} + \frac{69 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2048} \right) - \frac{59 \arctan\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{1024}}{\frac{(4 \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 1)^2}{d}}$	64

default	$\frac{4 \left(\frac{51 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{512} + \frac{69 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{2048} \right) - 59 \arctan \left(2 \tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{\left(4 \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 1 \right)^2} - \frac{59 \arctan \left(2 \tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{1024}$	64
risch	$-\frac{3i(59e^{3i(dx+c)} - 295e^{2i(dx+c)} + 241e^{i(dx+c)} - 45)}{256d(3e^{2i(dx+c)} - 10e^{i(dx+c)} + 3)^2} - \frac{59i \ln(e^{i(dx+c)} - 3)}{2048d} + \frac{59i \ln(e^{i(dx+c)} - \frac{1}{3})}{2048d}$	105

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-5+3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] `1/d*(-4*(51/512*tan(1/2*d*x+1/2*c)^3+69/2048*tan(1/2*d*x+1/2*c))/(4*tan(1/2*d*x+1/2*c)^2+1)^2-59/1024*arctan(2*tan(1/2*d*x+1/2*c))`

Maxima [A]

time = 0.51, size = 112, normalized size = 1.35

$$\frac{6 \left(\frac{23 \sin(dx+c)}{\cos(dx+c)+1} + \frac{68 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) + 59 \arctan \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} \right)}{\frac{8 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{16 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1} \cdot \frac{1}{1024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+3*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] `-1/1024*(6*(23*sin(d*x + c)/(cos(d*x + c) + 1) + 68*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 16*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1) + 59*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d`

Fricas [A]

time = 0.36, size = 90, normalized size = 1.08

$$\frac{59(9 \cos(dx+c)^2 - 30 \cos(dx+c) + 25) \arctan \left(\frac{5 \cos(dx+c) - 3}{4 \sin(dx+c)} \right) + 12(45 \cos(dx+c) - 91) \sin(dx+c)}{2048(9d \cos(dx+c)^2 - 30d \cos(dx+c) + 25d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+3*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] `1/2048*(59*(9*cos(d*x + c)^2 - 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c)) + 12*(45*cos(d*x + c) - 91)*sin(d*x + c))/(9*d*cos(d*x + c)^2 - 30*d*cos(d*x + c) + 25*d)`

Sympy [C] Result contains complex when optimal does not.

time = 1.47, size = 366, normalized size = 4.41

$$\left\{ \begin{array}{ll} \frac{(-5+3 \cosh(2 \operatorname{atanh}(\frac{1}{2})))^2}{5 \cos^2(\frac{1}{2})} & \text{for } c = -dx - 2i \operatorname{atanh}(\frac{1}{2}) \vee c = -dx + 2i \operatorname{atanh}(\frac{1}{2}) \\ \frac{944(\operatorname{atan}(2 \tan(\frac{1}{2} + \frac{1}{2}i)) + \frac{1}{2} \frac{1-2i}{1+i}) \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) - 472(\operatorname{atan}(2 \tan(\frac{1}{2} + \frac{1}{2}i)) + \frac{1}{2} \frac{1-2i}{1+i}) \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) - 59(\operatorname{atan}(2 \tan(\frac{1}{2} + \frac{1}{2}i)) + \frac{1}{2} \frac{1-2i}{1+i})}{16384i \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 8192 \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 1024} - \frac{472(\operatorname{atan}(2 \tan(\frac{1}{2} + \frac{1}{2}i)) + \frac{1}{2} \frac{1-2i}{1+i}) \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) - 59(\operatorname{atan}(2 \tan(\frac{1}{2} + \frac{1}{2}i)) + \frac{1}{2} \frac{1-2i}{1+i})}{16384i \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 8192 \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 1024} - \frac{488 \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i)}{16384i \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 8192 \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 1024} - \frac{138 \operatorname{Im}(\frac{1}{2} + \frac{1}{2}i)}{16384i \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 8192 \operatorname{Im}^2(\frac{1}{2} + \frac{1}{2}i) + 1024} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))**3,x)

[Out] Piecewise((x/(-5 + 3*cosh(2*atanh(1/2)))**3, Eq(c, -d*x - 2*I*atanh(1/2)) | Eq(c, -d*x + 2*I*atanh(1/2))), (x/(3*cos(c) - 5)**3, Eq(d, 0)), (-944*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) - 472*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) - 59*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) - 408*tan(c/2 + d*x/2)**3/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) - 138*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d), True))

Giac [A]

time = 0.44, size = 77, normalized size = 0.93

$$\frac{59 dx + 59 c + \frac{12 \left(68 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 23 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^2} - 118 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))^3,x, algorithm="giac")

[Out] -1/2048*(59*d*x + 59*c + 12*(68*tan(1/2*d*x + 1/2*c)^3 + 23*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 118*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d

Mupad [B]

time = 0.36, size = 96, normalized size = 1.16

$$\frac{59 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2} \right)}{1024 d} - \frac{59 \operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{1024 d} - \frac{\frac{51 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{2048} + \frac{69 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8192}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{2} + \frac{1}{16} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cos(c + d*x) - 5)^3,x)

[Out] (59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - (59*atan(2*tan(c/2 + (d*x)/2)))/(1024*d) - ((69*tan(c/2 + (d*x)/2))/8192 + (51*tan(c/2 + (d*x)/2)^3)/2048)/(d*(tan(c/2 + (d*x)/2)^2/2 + tan(c/2 + (d*x)/2)^4 + 1/16))

$$3.29 \quad \int \frac{1}{(-5+3\cos(c+dx))^4} dx$$

Optimal. Leaf size=108

$$\frac{385x}{32768} + \frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}$$

[Out] 385/32768*x+385/16384*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+1/16*sin(d*x+c)/d/(5-3*cos(d*x+c))^3+25/512*sin(d*x+c)/d/(5-3*cos(d*x+c))^2+311/8192*sin(d*x+c)/d/(5-3*cos(d*x+c))

Rubi [A]

time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 2833, 12, 2737}

$$\frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{385x}{32768}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 3*Cos[c + d*x])^(-4), x]

[Out] (385*x)/32768 + (385*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(16384*d) + Sin[c + d*x]/(16*d*(5 - 3*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(5 - 3*Cos[c + d*x])^2) + (311*Sin[c + d*x])/(8192*d*(5 - 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2737

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] & NegQ[a]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 + 3 \cos(c + dx))^4} dx &= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} - \frac{1}{48} \int \frac{15 + 6 \cos(c + dx)}{(-5 + 3 \cos(c + dx))^3} dx \\ &= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} + \frac{\int \frac{186 + 75 \cos(c + dx)}{(-5 + 3 \cos(c + dx))^2} dx}{1536} \\ &= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} + \frac{311 \sin(c + dx)}{8192d(5 - 3 \cos(c + dx))} \\ &= \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} + \frac{311 \sin(c + dx)}{8192d(5 - 3 \cos(c + dx))} \\ &= \frac{385x}{32768} + \frac{385 \tan^{-1}\left(\frac{\sin(c + dx)}{3 - \cos(c + dx)}\right)}{16384d} + \frac{\sin(c + dx)}{16d(5 - 3 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(5 - 3 \cos(c + dx))^2} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 66, normalized size = 0.61

$$\frac{770 \operatorname{ArcTan}\left(2 \tan\left(\frac{1}{2}(c + dx)\right)\right) - \frac{9(4883 \sin(c + dx) - 2340 \sin(2(c + dx)) + 311 \sin(3(c + dx)))}{(-5 + 3 \cos(c + dx))^3}}{32768d}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 3*Cos[c + d*x])^(-4), x]

[Out] (770*ArcTan[2*Tan[(c + d*x)/2]] - (9*(4883*Sin[c + d*x] - 2340*Sin[2*(c + d*x)] + 311*Sin[3*(c + d*x)]))/(-5 + 3*Cos[c + d*x])^3)/(32768*d)

Maple [A]

time = 0.09, size = 77, normalized size = 0.71

method	result
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derivativedivides	$\frac{\frac{369 \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{512} + \frac{117 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{256} + \frac{639 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{8192} + \frac{385 \arctan \left(2 \tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{16384}}{\left(4 \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 1 \right)^3} \cdot d}$
default	$\frac{\frac{369 \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{512} + \frac{117 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{256} + \frac{639 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{8192} + \frac{385 \arctan \left(2 \tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{16384}}{\left(4 \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 1 \right)^3} \cdot d}$
risch	$\frac{i(10395 e^{5i(dx+c)} - 86625 e^{4i(dx+c)} + 239470 e^{3i(dx+c)} - 218466 e^{2i(dx+c)} + 73575 e^{i(dx+c)} - 8397)}{12288d(3e^{2i(dx+c)} - 10e^{i(dx+c)} + 3)^3} + \frac{385i \ln(e^{i(dx+c)} - 3)}{32768d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-5+3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out] $1/d*(8*(369/4096*\tan(1/2*d*x+1/2*c))^5+117/2048*\tan(1/2*d*x+1/2*c)^3+639/65536*\tan(1/2*d*x+1/2*c))/(4*\tan(1/2*d*x+1/2*c)^2+1)^3+385/16384*\arctan(2*\tan(1/2*d*x+1/2*c))$

Maxima [A]

time = 0.49, size = 152, normalized size = 1.41

$$\frac{18 \left(\frac{71 \sin(dx+c)}{\cos(dx+c)+1} + \frac{416 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{656 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{\frac{12 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{48 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{64 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1} + 385 \arctan \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} \right)$$

$16384 d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+3*cos(d*x+c))^4,x, algorithm="maxima")`

[Out] $1/16384*(18*(71*\sin(d*x + c)/(\cos(d*x + c) + 1) + 416*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 656*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/(12*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 48*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 64*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6 + 1) + 385*\arctan(2*\sin(d*x + c)/(\cos(d*x + c) + 1)))/d$

Fricas [A]

time = 0.39, size = 121, normalized size = 1.12

$$\frac{385 (27 \cos(dx+c)^3 - 135 \cos(dx+c)^2 + 225 \cos(dx+c) - 125) \arctan\left(\frac{5 \cos(dx+c) - 3}{4 \sin(dx+c)}\right) + 36 (311 \cos(dx+c)^2 - 1170 \cos(dx+c) + 1143) \sin(dx+c)}{32768 (27 d \cos(dx+c)^3 - 135 d \cos(dx+c)^2 + 225 d \cos(dx+c) - 125 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+3*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] $-1/32768*(385*(27*\cos(d*x + c)^3 - 135*\cos(d*x + c)^2 + 225*\cos(d*x + c) - 125)*\arctan(1/4*(5*\cos(d*x + c) - 3)/\sin(d*x + c)) + 36*(311*\cos(d*x + c)^2 - 1170*\cos(d*x + c) + 1143)*\sin(d*x + c))/(27*d*\cos(d*x + c)^3 - 135*d*\cos(d*x + c)^2 + 225*d*\cos(d*x + c) - 125*d)$

Sympy [C] Result contains complex when optimal does not.

time = 3.13, size = 597, normalized size = 5.53



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))**4,x)

[Out] Piecewise((x/(-5 + 3*cosh(2*atanh(1/2)))**4, Eq(c, -d*x - 2*I*atanh(1/2)) | Eq(c, -d*x + 2*I*atanh(1/2))), (x/(3*cos(c) - 5)**4, Eq(d, 0)), (24640*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 18480*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 4620*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 385*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 11808*tan(c/2 + d*x/2)**5/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 7488*tan(c/2 + d*x/2)**3/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 1278*tan(c/2 + d*x/2)/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d), True))

Giac [A]

time = 0.45, size = 90, normalized size = 0.83

$$\frac{385 dx + 385 c + \frac{36 \left(656 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 416 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 71 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\left(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right)^3} - 770 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{32768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+3*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/32768*(385*d*x + 385*c + 36*(656*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x + 1/2*c)^3 + 71*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^3 - 770*arctan(sin(d*x + c)/(cos(d*x + c) - 3))/d

Mupad [B]

time = 0.00, size = 97, normalized size = 0.90

$$\frac{385 \operatorname{atan}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{16384 d} - \frac{385 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{16384 d} + \frac{\frac{369 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{512} + \frac{117 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{256} + \frac{639 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8192}}{d \left(4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(c + d*x) - 5)^4,x)
```

```
[Out] (385*atan(2*tan(c/2 + (d*x)/2)))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(16384*d) + ((639*tan(c/2 + (d*x)/2))/8192 + (117*tan(c/2 + (d*x)/2)^3)/256 + (369*tan(c/2 + (d*x)/2)^5)/512)/(d*(4*tan(c/2 + (d*x)/2)^2 + 1)^3)
```


$$3.30 \quad \int \frac{1}{-5-3\cos(c+dx)} dx$$

Optimal. Leaf size=31

$$-\frac{x}{4} + \frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

[Out] $-1/4*x+1/2*\arctan(\sin(d*x+c)/(3+\cos(d*x+c)))/d$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2737}

$$\frac{\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d} - \frac{x}{4}$$

Antiderivative was successfully verified.

[In] Int[(-5 - 3*Cos[c + d*x])^(-1), x]

[Out] $-1/4*x + \text{ArcTan}[\text{Sin}[c + d*x]/(3 + \text{Cos}[c + d*x])]/(2*d)$

Rule 2737

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] & NegQ[a]

Rubi steps

$$\int \frac{1}{-5-3\cos(c+dx)} dx = -\frac{x}{4} + \frac{\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 0.65

$$\frac{\text{ArcTan}\left(2 \cot\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 - 3*Cos[c + d*x])^(-1), x]

[Out] ArcTan[2*Cot[(c + d*x)/2]]/(2*d)

Maple [A]

time = 0.05, size = 18, normalized size = 0.58

method	result	size
derivativdivides	$\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{2d}$	18
risch	$-\frac{i \ln(e^{i(dx+c)}+3)}{4d} + \frac{i \ln(e^{i(dx+c)}+\frac{1}{3})}{4d}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-5-3*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/2/d*arctan(1/2*tan(1/2*d*x+1/2*c))

Maxima [A]

time = 0.48, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c)),x, algorithm="maxima")

[Out] -1/2*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1))/d

Fricas [A]

time = 0.38, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{5 \cos(dx+c)+3}{4 \sin(dx+c)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/4*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c))/d

Sympy [A]

time = 0.34, size = 44, normalized size = 1.42

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right) + \pi \left\lfloor \frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor}{2d} & \text{for } d \neq 0 \\ \frac{x}{-3 \cos(c) - 5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c)),x)

[Out] Piecewise((-atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(2*d), Ne(d, 0)), (x/(-3*cos(c) - 5), True))

Giac [A]

time = 0.43, size = 30, normalized size = 0.97

$$-\frac{dx + c - 2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c)),x, algorithm="giac")

[Out] -1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d

Mupad [B]

time = 0.25, size = 38, normalized size = 1.23

$$\frac{\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}}{2d} - \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(3*cos(c + d*x) + 5),x)

[Out] (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d) - atan(tan(c/2 + (d*x)/2)/2)/(2*d)

3.31 $\int \frac{1}{(-5-3\cos(c+dx))^2} dx$

Optimal. Leaf size=56

$$\frac{5x}{64} - \frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))}$$

[Out] 5/64*x-5/32*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-3/16*sin(d*x+c)/d/(5+3*cos(d*x+c))

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2743, 12, 2737}

$$-\frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{32d} - \frac{3\sin(c+dx)}{16d(3\cos(c+dx)+5)} + \frac{5x}{64}$$

Antiderivative was successfully verified.

[In] Int[(-5 - 3*Cos[c + d*x])^(-2), x]

[Out] (5*x)/64 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(32*d) - (3*Sin[c + d*x])/(16*d*(5 + 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2737

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] & NegQ[a]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 - 3 \cos(c + dx))^2} dx &= -\frac{3 \sin(c + dx)}{16d(5 + 3 \cos(c + dx))} - \frac{1}{16} \int \frac{5}{-5 - 3 \cos(c + dx)} dx \\ &= -\frac{3 \sin(c + dx)}{16d(5 + 3 \cos(c + dx))} - \frac{5}{16} \int \frac{1}{-5 - 3 \cos(c + dx)} dx \\ &= \frac{5x}{64} - \frac{5 \tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3 \sin(c + dx)}{16d(5 + 3 \cos(c + dx))} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 0.77

$$\frac{5 \operatorname{ArcTan}\left(2 \cot\left(\frac{1}{2}(c + dx)\right)\right) + \frac{6 \sin(c+dx)}{5+3 \cos(c+dx)}}{32d}$$

Antiderivative was successfully verified.

`[In] Integrate[(-5 - 3*Cos[c + d*x])^(-2), x]``[Out] -1/32*(5*ArcTan[2*Cot[(c + d*x)/2]] + (6*Sin[c + d*x])/(5 + 3*Cos[c + d*x]))/d`**Maple [A]**

time = 0.06, size = 46, normalized size = 0.82

method	result	size
derivativedivides	$\frac{-\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)} + \frac{5 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
default	$\frac{-\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)} + \frac{5 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
risch	$-\frac{i(5e^{i(dx+c)}+3)}{8d(3e^{2i(dx+c)}+10e^{i(dx+c)}+3)} + \frac{5i \ln(e^{i(dx+c)}+3)}{64d} - \frac{5i \ln(e^{i(dx+c)}+\frac{1}{3})}{64d}$	83

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-5-3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)``[Out] 1/d*(-3/16*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+4)+5/32*arctan(1/2*tan(1/2*d*x+1/2*c)))`**Maxima [A]**

time = 0.49, size = 68, normalized size = 1.21

$$\frac{\frac{6 \sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 4\right)(\cos(dx+c)+1)}}{32d} - 5 \arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))^2,x, algorithm="maxima")

[Out] -1/32*(6*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d

Fricas [A]

time = 0.35, size = 59, normalized size = 1.05

$$\frac{5(3 \cos(dx + c) + 5) \arctan\left(\frac{5 \cos(dx+c)+3}{4 \sin(dx+c)}\right) + 12 \sin(dx + c)}{64(3d \cos(dx + c) + 5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))^2,x, algorithm="fricas")

[Out] -1/64*(5*(3*cos(d*x + c) + 5)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 12*sin(d*x + c))/(3*d*cos(d*x + c) + 5*d)

Sympy [C] Result contains complex when optimal does not.

time = 0.88, size = 190, normalized size = 3.39

$$\begin{cases} \frac{x}{(-5-3 \cosh(2 \operatorname{atanh}(2)))^2} & \text{for } c = -dx - 2i \operatorname{atanh}(2) \vee c = -dx + 2i \operatorname{atanh}(2) \\ \frac{x}{(-3 \cos(c)-5)^2} & \text{for } d = 0 \\ \frac{5 \left(\operatorname{atan}\left(\frac{\tan\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right) + \pi \left\lfloor \frac{\frac{x}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor\right)}{32d \tan^2\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right) + 128d} + \frac{20 \left(\operatorname{atan}\left(\frac{\tan\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right) + \pi \left\lfloor \frac{\frac{x}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi} \right\rfloor\right)}{32d \tan^2\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right) + 128d} \right)}{32d \tan^2\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right) + 128d} - \frac{6 \tan\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right)}{32d \tan^2\left(\frac{\frac{x}{2} + \frac{dx}{2}}{2}\right) + 128d} \right)}{64d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))^2,x)

[Out] Piecewise((x/(-5 - 3*cosh(2*atanh(2)))**2, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c, -d*x + 2*I*atanh(2))), (x/(-3*cos(c) - 5)**2, Eq(d, 0)), (5*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(32*d*tan(c/2 + d*x/2)**2 + 128*d) + 20*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(32*d*tan(c/2 + d*x/2)**2 + 128*d) - 6*tan(c/2 + d*x/2)/(32*d*tan(c/2 + d*x/2)**2 + 128*d), True))

Giac [A]

time = 0.44, size = 59, normalized size = 1.05

$$\frac{5 dx + 5 c - \frac{12 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4} - 10 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (5 \cdot dx + 5 \cdot c - 12 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 + 4) - 10 \cdot \arctan(\sin(dx + c) / (\cos(dx + c) + 3)) / d$

Mupad [B]

time = 0.00, size = 67, normalized size = 1.20

$$\frac{5 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{32d} - \frac{5 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{32d} - \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(c + d*x) + 5)^2,x)`

[Out] $(5 \cdot \operatorname{atan}(\tan(c/2 + (d \cdot x)/2)/2)) / (32 \cdot d) - (5 \cdot (\operatorname{atan}(\tan(c/2 + (d \cdot x)/2)) - (d \cdot x)/2)) / (32 \cdot d) - (3 \cdot \tan(c/2 + (d \cdot x)/2)) / (16 \cdot d \cdot (\tan(c/2 + (d \cdot x)/2)^2 + 4))$

$$3.32 \quad \int \frac{1}{(-5-3\cos(c+dx))^3} dx$$

Optimal. Leaf size=81

$$-\frac{59x}{2048} + \frac{59\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d} + \frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}$$

[Out] -59/2048*x+59/1024*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d+3/32*sin(d*x+c)/d/(5+3*cos(d*x+c))^2+45/512*sin(d*x+c)/d/(5+3*cos(d*x+c))

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {2743, 2833, 12, 2737}

$$\frac{59\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1024d} + \frac{45\sin(c+dx)}{512d(3\cos(c+dx)+5)} + \frac{3\sin(c+dx)}{32d(3\cos(c+dx)+5)^2} - \frac{59x}{2048}$$

Antiderivative was successfully verified.

[In] Int[(-5 - 3*Cos[c + d*x])^(-3), x]

[Out] (-59*x)/2048 + (59*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(1024*d) + (3*Sin[c + d*x])/(32*d*(5 + 3*Cos[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(5 + 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2737

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && NegQ[a]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 - 3 \cos(c + dx))^3} dx &= \frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} - \frac{1}{32} \int \frac{10 - 3 \cos(c + dx)}{(-5 - 3 \cos(c + dx))^2} dx \\ &= \frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))} + \frac{1}{512} \int \frac{59}{-5 - 3 \cos(c + dx)} dx \\ &= \frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))} + \frac{59}{512} \int \frac{1}{-5 - 3 \cos(c + dx)} dx \\ &= -\frac{59x}{2048} + \frac{59 \tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d} + \frac{3 \sin(c + dx)}{32d(5 + 3 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 65, normalized size = 0.80

$$\frac{59 \operatorname{ArcTan}\left(2 \cot\left(\frac{1}{2}(c + dx)\right)\right) (5 + 3 \cos(c + dx))^2 + 546 \sin(c + dx) + 135 \sin(2(c + dx))}{1024d(5 + 3 \cos(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 - 3*Cos[c + d*x])^(-3), x]

[Out] (59*ArcTan[2*Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 546*Sin[c + d*x] + 135*Sin[2*(c + d*x)])/(1024*d*(5 + 3*Cos[c + d*x])^2)

Maple [A]

time = 0.08, size = 62, normalized size = 0.77

method	result	size
derivativedivides	$\frac{-\frac{69 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{128} - \frac{51 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{32} - \frac{59 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2}\right)}{1024}}{4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\right)^2} d$	62

default	$-\frac{69 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 51 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{128 \cdot 4 \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) + 4 \right)^2} - \frac{59 \arctan \left(\frac{\tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{2} \right)}{1024}$	62
risch	$\frac{3i(59e^{3i(dx+c)} + 295e^{2i(dx+c)} + 241e^{i(dx+c)} + 45)}{256d(3e^{2i(dx+c)} + 10e^{i(dx+c)} + 3)^2} + \frac{59i \ln(e^{i(dx+c)} + \frac{1}{3})}{2048d} - \frac{59i \ln(e^{i(dx+c)} + 3)}{2048d}$	105

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-5-3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] $1/d \cdot (-1/4 \cdot (-69/128 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c))^3 - 51/32 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 4)^2 - 59/1024 \cdot \arctan(1/2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c))$

Maxima [A]

time = 0.59, size = 111, normalized size = 1.37

$$\frac{6 \left(\frac{68 \sin(dx+c)}{\cos(dx+c)+1} + \frac{23 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{\frac{8 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 16} - 59 \arctan \left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)} \right)$$

1024 d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5-3*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $1/1024 \cdot (6 \cdot (68 \cdot \sin(dx+c) / (\cos(dx+c)+1) + 23 \cdot \sin(dx+c)^3 / (\cos(dx+c)+1)^3) / (8 \cdot \sin(dx+c)^2 / (\cos(dx+c)+1)^2 + \sin(dx+c)^4 / (\cos(dx+c)+1)^4 + 16) - 59 \cdot \arctan(1/2 \cdot \sin(dx+c) / (\cos(dx+c)+1))) / d$

Fricas [A]

time = 0.38, size = 90, normalized size = 1.11

$$\frac{59(9 \cos(dx+c)^2 + 30 \cos(dx+c) + 25) \arctan \left(\frac{5 \cos(dx+c) + 3}{4 \sin(dx+c)} \right) + 12(45 \cos(dx+c) + 91) \sin(dx+c)}{2048(9d \cos(dx+c)^2 + 30d \cos(dx+c) + 25d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5-3*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $1/2048 \cdot (59 \cdot (9 \cdot \cos(dx+c)^2 + 30 \cdot \cos(dx+c) + 25) \cdot \arctan(1/4 \cdot (5 \cdot \cos(dx+c) + 3) / \sin(dx+c)) + 12 \cdot (45 \cdot \cos(dx+c) + 91) \cdot \sin(dx+c)) / (9 \cdot d \cdot \cos(dx+c)^2 + 30 \cdot d \cdot \cos(dx+c) + 25 \cdot d)$

Sympy [C] Result contains complex when optimal does not.

time = 1.55, size = 362, normalized size = 4.47

$$\frac{\frac{(-5-3 \cosh(2 \operatorname{atanh}(2)))^2}{(-3 \cos(c)-5)^2}}{1024d \tan^2\left(\frac{x}{2} + \frac{c}{2}\right) + 8192d \tan^4\left(\frac{x}{2} + \frac{c}{2}\right) + 10384d} - \frac{472 \left(\operatorname{atan} \left(\frac{\sin \left(\frac{x}{2} + \frac{c}{2} \right)}{\cos \left(\frac{x}{2} + \frac{c}{2} \right)} \right) + \frac{21 \cos \left(\frac{x}{2} + \frac{c}{2} \right)}{\cos \left(\frac{x}{2} + \frac{c}{2} \right)} \right) \tan^2 \left(\frac{x}{2} + \frac{c}{2} \right)}{1024d \tan^2\left(\frac{x}{2} + \frac{c}{2}\right) + 8192d \tan^4\left(\frac{x}{2} + \frac{c}{2}\right) + 10384d} - \frac{944 \left(\operatorname{atan} \left(\frac{\sin \left(\frac{x}{2} + \frac{c}{2} \right)}{\cos \left(\frac{x}{2} + \frac{c}{2} \right)} \right) + \frac{21 \cos \left(\frac{x}{2} + \frac{c}{2} \right)}{\cos \left(\frac{x}{2} + \frac{c}{2} \right)} \right)}{1024d \tan^2\left(\frac{x}{2} + \frac{c}{2}\right) + 8192d \tan^4\left(\frac{x}{2} + \frac{c}{2}\right) + 10384d} + \frac{138 \tan^2\left(\frac{x}{2} + \frac{c}{2}\right)}{1024d \tan^2\left(\frac{x}{2} + \frac{c}{2}\right) + 8192d \tan^4\left(\frac{x}{2} + \frac{c}{2}\right) + 10384d} + \frac{408 \tan\left(\frac{x}{2} + \frac{c}{2}\right)}{1024d \tan^2\left(\frac{x}{2} + \frac{c}{2}\right) + 8192d \tan^4\left(\frac{x}{2} + \frac{c}{2}\right) + 10384d}$$

for $c = -dx - 2i \operatorname{atanh}(2) \vee c = -dx + 2i \operatorname{atanh}(2)$
for $d = 0$
otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))**3,x)

[Out] Piecewise((x/(-5 - 3*cosh(2*atanh(2)))**3, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c, -d*x + 2*I*atanh(2))), (x/(-3*cos(c) - 5)**3, Eq(d, 0)), (-59*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) - 472*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) - 944*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 138*tan(c/2 + d*x/2)**3/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 408*tan(c/2 + d*x/2)/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d), True))

Giac [A]

time = 0.44, size = 75, normalized size = 0.93

$$\frac{59 dx + 59 c - \frac{12 \left(23 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 68 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4 \right)^2} - 118 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))^3,x, algorithm="giac")

[Out] -1/2048*(59*d*x + 59*c - 12*(23*tan(1/2*d*x + 1/2*c)^3 + 68*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 118*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d

Mupad [B]

time = 0.36, size = 95, normalized size = 1.17

$$\frac{59 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2} \right)}{1024 d} - \frac{59 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{1024 d} + \frac{\frac{69 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{512} + \frac{51 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{128}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 8 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 16 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(3*cos(c + d*x) + 5)^3,x)

[Out] (59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - (59*atan(tan(c/2 + (d*x)/2)/2))/(1024*d) + ((51*tan(c/2 + (d*x)/2))/128 + (69*tan(c/2 + (d*x)/2)^3)/512)/(d*(8*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 16))

3.33 $\int \frac{1}{(-5-3\cos(c+dx))^4} dx$

Optimal. Leaf size=106

$$\frac{385x}{32768} - \frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{16384d} - \frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}$$

[Out] 385/32768*x-385/16384*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-1/16*sin(d*x+c)/d/(5+3*cos(d*x+c))^3-25/512*sin(d*x+c)/d/(5+3*cos(d*x+c))^2-311/8192*sin(d*x+c)/d/(5+3*cos(d*x+c))

Rubi [A]

time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 2833, 12, 2737}

$$-\frac{385\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{16384d} - \frac{311\sin(c+dx)}{8192d(3\cos(c+dx)+5)} - \frac{25\sin(c+dx)}{512d(3\cos(c+dx)+5)^2} - \frac{\sin(c+dx)}{16d(3\cos(c+dx)+5)^3} + \frac{385x}{32768}$$

Antiderivative was successfully verified.

[In] Int[(-5 - 3*Cos[c + d*x])^(-4), x]

[Out] (385*x)/32768 - (385*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(16384*d) - Sin[c + d*x]/(16*d*(5 + 3*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(5 + 3*Cos[c + d*x])^2) - (311*Sin[c + d*x])/(8192*d*(5 + 3*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2737

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a - q + b*Sin[c + d*x]))], x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] & & NegQ[a]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 - 3 \cos(c + dx))^4} dx &= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{1}{48} \int \frac{15 - 6 \cos(c + dx)}{(-5 - 3 \cos(c + dx))^3} dx \\ &= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} + \frac{\int \frac{186 - 75 \cos(c + dx)}{(-5 - 3 \cos(c + dx))^2} dx}{1536} \\ &= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} - \frac{311 \sin(c + dx)}{8192d(5 + 3 \cos(c + dx))} \\ &= -\frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} - \frac{311 \sin(c + dx)}{8192d(5 + 3 \cos(c + dx))} \\ &= \frac{385x}{32768} - \frac{385 \tan^{-1}\left(\frac{\sin(c + dx)}{3 + \cos(c + dx)}\right)}{16384d} - \frac{\sin(c + dx)}{16d(5 + 3 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(5 + 3 \cos(c + dx))^2} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 66, normalized size = 0.62

$$-\frac{770 \operatorname{ArcTan}\left(2 \cot\left(\frac{1}{2}(c + dx)\right)\right) + \frac{9(4883 \sin(c + dx) + 2340 \sin(2(c + dx)) + 311 \sin(3(c + dx)))}{(5 + 3 \cos(c + dx))^3}}{32768d}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 - 3*Cos[c + d*x])^(-4), x]

[Out] -1/32768*(770*ArcTan[2*Cot[(c + d*x)/2]] + (9*(4883*Sin[c + d*x] + 2340*Sin[2*(c + d*x)] + 311*Sin[3*(c + d*x)]))/(5 + 3*Cos[c + d*x])^3)/d

Maple [A]

time = 0.09, size = 75, normalized size = 0.71

method	result
--------	--------

derivativdivides	$\frac{-\frac{639\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{369\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}+\frac{385\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3} \frac{1}{d}$
default	$\frac{-\frac{639\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{369\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}+\frac{385\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3} \frac{1}{d}$
risch	$-\frac{i(10395e^{5i(dx+c)}+86625e^{4i(dx+c)}+239470e^{3i(dx+c)}+218466e^{2i(dx+c)}+73575e^{i(dx+c)}+8397)}{12288d(3e^{2i(dx+c)}+10e^{i(dx+c)}+3)^3}-\frac{385i\ln(e^{i(dx+c)}+3)}{32768d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-5-3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out] `1/d*(1/8*(-639/1024*tan(1/2*d*x+1/2*c)^5-117/32*tan(1/2*d*x+1/2*c)^3-369/64*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^3+385/16384*arctan(1/2*tan(1/2*d*x+1/2*c))`

Maxima [A]

time = 0.50, size = 151, normalized size = 1.42

$$\frac{18\left(\frac{656\sin(dx+c)}{\cos(dx+c)+1}+\frac{416\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{71\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)-385\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64} \frac{1}{16384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5-3*cos(d*x+c))^4,x, algorithm="maxima")`

[Out] `-1/16384*(18*(656*sin(d*x + c)/(cos(d*x + c) + 1) + 416*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 71*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 385*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d`

Fricas [A]

time = 0.36, size = 121, normalized size = 1.14

$$-\frac{385\left(27\cos(dx+c)^3+135\cos(dx+c)^2+225\cos(dx+c)+125\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)+36\left(311\cos(dx+c)^2+1170\cos(dx+c)+1143\right)\sin(dx+c)}{32768\left(27d\cos(dx+c)^3+135d\cos(dx+c)^2+225d\cos(dx+c)+125d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5-3*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] `-1/32768*(385*(27*cos(d*x + c)^3 + 135*cos(d*x + c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 36*(311*cos(d*x + c)^2`

+ 1170*cos(d*x + c) + 1143)*sin(d*x + c))/(27*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)

Sympy [C] Result contains complex when optimal does not.

time = 3.28, size = 595, normalized size = 5.61



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))**4,x)

[Out] Piecewise((x/(-5 - 3*cosh(2*atanh(2)))**4, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c, -d*x + 2*I*atanh(2))), (x/(-3*cos(c) - 5)**4, Eq(d, 0)), (385*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 4620*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 18480*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 24640*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 1278*tan(c/2 + d*x/2)**5/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 7488*tan(c/2 + d*x/2)**3/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 11808*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d), True))

Giac [A]

time = 0.44, size = 88, normalized size = 0.83

$$\frac{385 dx + 385 c - \frac{36 \left(71 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 416 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 656 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4 \right)^3} - 770 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3} \right)}{32768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5-3*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/32768*(385*d*x + 385*c - 36*(71*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x + 1/2*c)^3 + 656*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^3 - 770*arctan(sin(d*x + c)/(cos(d*x + c) + 3))/d

Mupad [B]

time = 0.00, size = 96, normalized size = 0.91

$$\frac{385 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{16384 d} - \frac{385 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{16384 d} - \frac{\frac{639 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8192} + \frac{117 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{256} + \frac{369 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{512}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(c + d*x) + 5)^4,x)`

[Out] `(385*atan(tan(c/2 + (d*x)/2)/2))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2) - (d*x)/2))/(16384*d) - ((369*tan(c/2 + (d*x)/2))/512 + (117*tan(c/2 + (d*x)/2)^3)/256 + (639*tan(c/2 + (d*x)/2)^5)/8192)/(d*(tan(c/2 + (d*x)/2)^2 + 4)^3)`

3.34 $\int \frac{1}{3+5 \cos(c+dx)} dx$

Optimal. Leaf size=65

$$-\frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

[Out] $-1/4*\ln(2*\cos(1/2*d*x+1/2*c)-\sin(1/2*d*x+1/2*c))/d+1/4*\ln(2*\cos(1/2*d*x+1/2*c)+\sin(1/2*d*x+1/2*c))/d$

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 212}

$$\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right) + 2 \cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3 + 5*\text{Cos}[c + d*x])^{-1}, x]$

[Out] $-1/4*\text{Log}[2*\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]]/d + \text{Log}[2*\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]]/(4*d)$

Rule 212

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] \text{ /; } \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{Gt} \text{Q}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2738

$\text{Int}[(a_ + (b_)*\sin[\text{Pi}/2 + (c_ + (d_)*(x_))])^{-1}, x_Symbol] \text{ :> } \text{With}\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[2*(e/d), \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{3+5 \cos(c+dx)} dx &= \frac{2\text{Subst}\left(\int \frac{1}{8-2x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} \\ &= -\frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 65, normalized size = 1.00

$$-\frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 + 5*Cos[c + d*x])^(-1),x]`

```
[Out] -1/4*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/d + Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(4*d)
```

Maple [A]

time = 0.05, size = 34, normalized size = 0.52

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{4} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{4}}{d}$	34
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{4} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{4}}{d}$	34
norman	$-\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{4d} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{4d}$	36
risch	$\frac{\ln\left(e^{i(dx+c)} + \frac{3}{5} + \frac{4i}{5}\right)}{4d} - \frac{\ln\left(e^{i(dx+c)} + \frac{3}{5} - \frac{4i}{5}\right)}{4d}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(3+5*cos(d*x+c)),x,method=_RETURNVERBOSE)`

```
[Out] 1/d*(1/4*ln(tan(1/2*d*x+1/2*c)+2)-1/4*ln(tan(1/2*d*x+1/2*c)-2))
```

Maxima [A]

time = 0.28, size = 48, normalized size = 0.74

$$\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2\right) - \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(3+5*cos(d*x+c)),x, algorithm="maxima")`

```
[Out] 1/4*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Fricas [A]

time = 0.35, size = 46, normalized size = 0.71

$$\frac{\log\left(\frac{3}{2}\cos(dx+c) + 2\sin(dx+c) + \frac{5}{2}\right) - \log\left(\frac{3}{2}\cos(dx+c) - 2\sin(dx+c) + \frac{5}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c)),x, algorithm="fricas")`

[Out] $\frac{1}{8} * (\log(3/2 * \cos(d*x + c) + 2 * \sin(d*x + c) + 5/2) - \log(3/2 * \cos(d*x + c) - 2 * \sin(d*x + c) + 5/2)) / d$

Sympy [A]

time = 0.32, size = 41, normalized size = 0.63

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 2\right)}{4d} + \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 2\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{5\cos(c)+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c)),x)`

[Out] `Piecewise((-log(tan(c/2 + d*x/2) - 2)/(4*d) + log(tan(c/2 + d*x/2) + 2)/(4*d), Ne(d, 0)), (x/(5*cos(c) + 3), True))`

Giac [A]

time = 0.41, size = 34, normalized size = 0.52

$$\frac{\log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2\right|\right) - \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2\right|\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c)),x, algorithm="giac")`

[Out] $\frac{1}{4} * (\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 2)) - \log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 2))) / d$

Mupad [B]

time = 0.36, size = 17, normalized size = 0.26

$$\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*cos(c + d*x) + 3),x)`

[Out] `atanh(tan(c/2 + (d*x)/2)/2)/(2*d)`

3.35 $\int \frac{1}{(3+5 \cos(c+dx))^2} dx$

Optimal. Leaf size=90

$$\frac{3 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right)}{64d} - \frac{3 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) + \sin \left(\frac{1}{2}(c+dx) \right) \right)}{64d} + \frac{5 \sin(c+dx)}{16d(3+5 \cos(c+dx))}$$

[Out] 3/64*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/64*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/16*sin(d*x+c)/d/(3+5*cos(d*x+c))

Rubi [A]

time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {2743, 12, 2738, 212}

$$\frac{5 \sin(c+dx)}{16d(5 \cos(c+dx)+3)} + \frac{3 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right)}{64d} - \frac{3 \log \left(\sin \left(\frac{1}{2}(c+dx) \right) + 2 \cos \left(\frac{1}{2}(c+dx) \right) \right)}{64d}$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*Cos[c + d*x])^(-2), x]

[Out] (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(64*d) - (3*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(64*d) + (5*Sin[c + d*x])/(16*d*(3 + 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist

$[1/((n + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Simp}[a*(n + 1) - b*(n + 2)*\text{Sin}[c + d*x], x], x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(3 + 5 \cos(c + dx))^2} dx &= \frac{5 \sin(c + dx)}{16d(3 + 5 \cos(c + dx))} + \frac{1}{16} \int -\frac{3}{3 + 5 \cos(c + dx)} dx \\ &= \frac{5 \sin(c + dx)}{16d(3 + 5 \cos(c + dx))} - \frac{3}{16} \int \frac{1}{3 + 5 \cos(c + dx)} dx \\ &= \frac{5 \sin(c + dx)}{16d(3 + 5 \cos(c + dx))} - \frac{3 \text{Subst}\left(\int \frac{1}{8-2x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{8d} \\ &= \frac{3 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d} - \frac{3 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 143, normalized size = 1.59

$$\frac{9 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 15 \cos(c + dx) \left(\log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) - 9 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right) + 20 \sin(c + dx)}{64d(3 + 5 \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 5*Cos[c + d*x])^(-2), x]

[Out] (9*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 15*Cos[c + d*x]*(Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d*(3 + 5*Cos[c + d*x]))

Maple [A]

time = 0.06, size = 64, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-\frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{64} - \frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{64}}{d}$	64
default	$\frac{-\frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{64} - \frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{64}}{d}$	64
norman	$-\frac{5 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16d \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 4\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{64d} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{64d}$	65

risch	$\frac{i(3e^{i(dx+c)}+5)}{8d(5e^{2i(dx+c)}+6e^{i(dx+c)}+5)} - \frac{3\ln(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5})}{64d} + \frac{3\ln(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5})}{64d}$	85
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3+5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out] $1/d*(-5/32/(\tan(1/2*d*x+1/2*c)+2)-3/64*\ln(\tan(1/2*d*x+1/2*c)+2)-5/32/(\tan(1/2*d*x+1/2*c)-2)+3/64*\ln(\tan(1/2*d*x+1/2*c)-2))$

Maxima [A]

time = 0.28, size = 91, normalized size = 1.01

$$\frac{\frac{20 \sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)} + 3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2\right) - 3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c))^2,x, algorithm="maxima")`

[Out] $-1/64*(20*\sin(d*x + c)/((\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 - 4)*(\cos(d*x + c) + 1)) + 3*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 2) - 3*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 2))/d$

Fricas [A]

time = 0.38, size = 88, normalized size = 0.98

$$\frac{3(5 \cos(dx+c)+3) \log\left(\frac{3}{2} \cos(dx+c)+2 \sin(dx+c)+\frac{5}{2}\right) - 3(5 \cos(dx+c)+3) \log\left(\frac{3}{2} \cos(dx+c)-2 \sin(dx+c)+\frac{5}{2}\right) - 40 \sin(dx+c)}{128(5d \cos(dx+c)+3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] $-1/128*(3*(5*\cos(d*x + c) + 3)*\log(3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - 3*(5*\cos(d*x + c) + 3)*\log(3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) - 40*\sin(d*x + c))/(5*d*\cos(d*x + c) + 3*d)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(78) = 156.

time = 0.67, size = 228, normalized size = 2.53

$$\left\{ \begin{array}{ll} \frac{\frac{x}{(5 \cos(2 \operatorname{atan}(2))+3)^2}}{(5 \cos(c)+3)^2} & \text{for } c = -dx - 2 \operatorname{atan}(2) \vee c = -dx + 2 \operatorname{atan}(2) \\ \frac{3 \log\left(\tan\left(\frac{x}{5} + \frac{dx}{2}\right) - 2\right) \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right)}{64d \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right) - 256d} - \frac{12 \log\left(\tan\left(\frac{x}{5} + \frac{dx}{2}\right) - 2\right)}{64d \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right) - 256d} - \frac{3 \log\left(\tan\left(\frac{x}{5} + \frac{dx}{2}\right) + 2\right) \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right)}{64d \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right) - 256d} + \frac{12 \log\left(\tan\left(\frac{x}{5} + \frac{dx}{2}\right) + 2\right)}{64d \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right) - 256d} - \frac{20 \tan\left(\frac{x}{5} + \frac{dx}{2}\right)}{64d \tan^2\left(\frac{x}{5} + \frac{dx}{2}\right) - 256d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c))**2,x)`

[Out] Piecewise((x/(5*cos(2*atan(2)) + 3)**2, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*x + 2*atan(2))), (x/(5*cos(c) + 3)**2, Eq(d, 0)), (3*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 12*log(tan(c/2 + d*x/2) - 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 3*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) + 12*log(tan(c/2 + d*x/2) + 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 20*tan(c/2 + d*x/2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d), True))

Giac [A]

time = 0.46, size = 62, normalized size = 0.69

$$\frac{20 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 4} + 3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2\right|\right) - 3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2\right|\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 4) + 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) - 3*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d

Mupad [B]

time = 0.32, size = 47, normalized size = 0.52

$$-\frac{3 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{32 d} - \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*cos(c + d*x) + 3)^2,x)

[Out] - (3*atanh(tan(c/2 + (d*x)/2)/2))/(32*d) - (5*tan(c/2 + (d*x)/2))/(16*d*(tan(c/2 + (d*x)/2)^2 - 4))

3.36 $\int \frac{1}{(3+5 \cos(c+dx))^3} dx$

Optimal. Leaf size=115

$$-\frac{43 \log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5 \sin(c+dx)}{32d(3+5 \cos(c+dx))}$$

[Out] -43/2048*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d+43/2048*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/32*sin(d*x+c)/d/(3+5*cos(d*x+c))^2-45/512*sin(d*x+c)/d/(3+5*cos(d*x+c))

Rubi [A]

time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$,

Rules used = {2743, 2833, 12, 2738, 212}

$$-\frac{45 \sin(c+dx)}{512d(5 \cos(c+dx)+3)} + \frac{5 \sin(c+dx)}{32d(5 \cos(c+dx)+3)^2} - \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43 \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + 2 \cos\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*Cos[c + d*x])^(-3), x]

[Out] (-43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(2048*d) + (43*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(2048*d) + (5*Sin[c + d*x])/(32*d*(3 + 5*Cos[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist


```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(3 + 5 \cos(c + dx))^3} dx &= \frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} + \frac{1}{32} \int \frac{-6 + 5 \cos(c + dx)}{(3 + 5 \cos(c + dx))^2} dx \\
 &= \frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))} + \frac{1}{512} \int \frac{43}{3 + 5 \cos(c + dx)} dx \\
 &= \frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))} + \frac{43}{512} \int \frac{1}{3 + 5 \cos(c + dx)} dx \\
 &= \frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))} + \frac{43 \text{Subst}\left(\int \frac{1}{8-2x^2} dx, x, t\right)}{256d} \\
 &= -\frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} + \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d}
 \end{aligned}$$

Mathematica [A]

time = 0.13, size = 217, normalized size = 1.89

$$\frac{-\frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} + \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d}}{512d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} - \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{2048d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} - \frac{5}{512d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} - \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{2048d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (-43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(2048*d) + (43*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(2048*d) + 5/(512*d*(2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - 5/(512*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]

time = 0.08, size = 94, normalized size = 0.82

method	result
norman	$-\frac{35 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{85 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512d}}{128d \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 4\right)^2} - \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{2048d} + \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{2048d}$
derivativedivides	$-\frac{25}{512 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} + \frac{85}{1024 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} + \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{2048} + \frac{25}{512 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^2} + \frac{85}{1024 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} - \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{2048}$
default	$-\frac{25}{512 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} + \frac{85}{1024 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} + \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{2048} + \frac{25}{512 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^2} + \frac{85}{1024 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} - \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{2048}$
risch	$-\frac{i(215 e^{3i(dx+c)} + 387 e^{2i(dx+c)} + 325 e^{i(dx+c)} + 225)}{256d(5 e^{2i(dx+c)} + 6 e^{i(dx+c)} + 5)^2} + \frac{43 \ln\left(e^{i(dx+c)} + \frac{3}{5} + \frac{4i}{5}\right)}{2048d} - \frac{43 \ln\left(e^{i(dx+c)} + \frac{3}{5} - \frac{4i}{5}\right)}{2048d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3+5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] $1/d * (-25/512 / (\tan(1/2*d*x+1/2*c)+2)^2 + 85/1024 / (\tan(1/2*d*x+1/2*c)+2) + 43/2048 * \ln(\tan(1/2*d*x+1/2*c)+2) + 25/512 / (\tan(1/2*d*x+1/2*c)-2)^2 + 85/1024 / (\tan(1/2*d*x+1/2*c)-2) - 43/2048 * \ln(\tan(1/2*d*x+1/2*c)-2))$

Maxima [A]

time = 0.29, size = 135, normalized size = 1.17

$$\frac{20 \left(\frac{28 \sin(dx+c)}{\cos(dx+c)+1} - \frac{17 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{\frac{8 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 16} + 43 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2 \right) - 43 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2 \right)$$

2048 d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $1/2048 * (20 * (28 * \sin(d*x + c) / (\cos(d*x + c) + 1) - 17 * \sin(d*x + c)^3 / (\cos(d*x + c) + 1)^3) / (8 * \sin(d*x + c)^2 / (\cos(d*x + c) + 1)^2 - \sin(d*x + c)^4 / (\cos(d*x + c) + 1)^4 - 16) + 43 * \log(\sin(d*x + c) / (\cos(d*x + c) + 1) + 2) - 43 * \log(\sin(d*x + c) / (\cos(d*x + c) + 1) - 2)) / d$

Fricas [A]

time = 0.36, size = 129, normalized size = 1.12

$$\frac{43(25 \cos(dx+c)^2 + 30 \cos(dx+c) + 9) \log\left(\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 43(25 \cos(dx+c)^2 + 30 \cos(dx+c) + 9) \log\left(\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) - 40(45 \cos(dx+c) + 11) \sin(dx+c)}{4096(25d \cos(dx+c)^2 + 30d \cos(dx+c) + 9d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $1/4096 * (43 * (25 * \cos(d*x + c)^2 + 30 * \cos(d*x + c) + 9) * \log(3/2 * \cos(d*x + c) + 2 * \sin(d*x + c) + 5/2) - 43 * (25 * \cos(d*x + c)^2 + 30 * \cos(d*x + c) + 9) * \log(3$

$/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) - 40*(45*\cos(d*x + c) + 11)*\sin(d*x + c))/(25*d*\cos(d*x + c)^2 + 30*d*\cos(d*x + c) + 9*d)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(102) = 204$.

time = 1.24, size = 474, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))**3,x)

[Out] Piecewise((x/(5*cos(2*atan(2)) + 3)**3, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*x + 2*atan(2))), (x/(5*cos(c) + 3)**3, Eq(d, 0)), (-43*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**4/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 344*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 688*log(tan(c/2 + d*x/2) - 2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 43*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 344*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 688*log(tan(c/2 + d*x/2) + 2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 340*tan(c/2 + d*x/2)**3/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 560*tan(c/2 + d*x/2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d), True))

Giac [A]

time = 0.54, size = 78, normalized size = 0.68

$$\frac{20 \left(17 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 28 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 4 \right)^2} + 43 \log \left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2 \right| \right) - 43 \log \left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2 \right| \right)$$

$$2048 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))^3,x, algorithm="giac")

[Out] $1/2048*(20*(17*\tan(1/2*d*x + 1/2*c)^3 - 28*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 4)^2 + 43*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 2)) - 43*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 2)))/d$

Mupad [B]

time = 0.67, size = 76, normalized size = 0.66

$$\frac{43 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{1024 d} - \frac{\frac{35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{128} - \frac{85 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{512}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 8 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 16 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cos(c + d*x) + 3)^3,x)
```

```
[Out] (43*atanh(tan(c/2 + (d*x)/2)/2))/(1024*d) - ((35*tan(c/2 + (d*x)/2))/128 -  
(85*tan(c/2 + (d*x)/2)^3)/512)/(d*(tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)  
/2)^2 + 16))
```

$$3.37 \quad \int \frac{1}{(3+5 \cos(c+dx))^4} dx$$

Optimal. Leaf size=140

$$\frac{279 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right)}{32768d} - \frac{279 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) + \sin \left(\frac{1}{2}(c+dx) \right) \right)}{32768d} + \frac{5 \sin(c)}{48d(3+5 \cos(c+dx))}$$

[Out] 279/32768*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-279/32768*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/48*sin(d*x+c)/d/(3+5*cos(d*x+c))^3-25/512*sin(d*x+c)/d/(3+5*cos(d*x+c))^2+995/24576*sin(d*x+c)/d/(3+5*cos(d*x+c))

Rubi [A]

time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 212}

$$\frac{995 \sin(c+dx)}{24576d(5 \cos(c+dx)+3)} - \frac{25 \sin(c+dx)}{512d(5 \cos(c+dx)+3)^2} + \frac{5 \sin(c+dx)}{48d(5 \cos(c+dx)+3)^3} + \frac{279 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right)}{32768d} - \frac{279 \log \left(\sin \left(\frac{1}{2}(c+dx) \right) + 2 \cos \left(\frac{1}{2}(c+dx) \right) \right)}{32768d}$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*Cos[c + d*x])^(-4), x]

[Out] (279*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(32768*d) - (279*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(32768*d) + (5*Sin[c + d*x])/(48*d*(3 + 5*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x])^2) + (995*Sin[c + d*x])/(24576*d*(3 + 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(3 + 5 \cos(c + dx))^4} dx &= \frac{5 \sin(c + dx)}{48d(3 + 5 \cos(c + dx))^3} + \frac{1}{48} \int \frac{-9 + 10 \cos(c + dx)}{(3 + 5 \cos(c + dx))^3} dx \\ &= \frac{5 \sin(c + dx)}{48d(3 + 5 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))^2} + \frac{\int \frac{154 - 75 \cos(c + dx)}{(3 + 5 \cos(c + dx))^2} dx}{1536} \\ &= \frac{5 \sin(c + dx)}{48d(3 + 5 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))^2} + \frac{995 \sin(c + dx)}{24576d(3 + 5 \cos(c + dx))} \\ &= \frac{5 \sin(c + dx)}{48d(3 + 5 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))^2} + \frac{995 \sin(c + dx)}{24576d(3 + 5 \cos(c + dx))} \\ &= \frac{5 \sin(c + dx)}{48d(3 + 5 \cos(c + dx))^3} - \frac{25 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))^2} + \frac{995 \sin(c + dx)}{24576d(3 + 5 \cos(c + dx))} \\ &= \frac{279 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{32768d} - \frac{279 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{32768d} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 296 vs. 2(140) = 280.

time = 0.20, size = 296, normalized size = 2.11

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cos[c + d*x])^(-4), x]
```

```
[Out] (467046*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 104625*Cos[3*(c + d*x)]
)*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 765855*Cos[c + d*x]*(Log[2*C
```

os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 376650*Cos[2*(c + d*x)]*(Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 467046*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 226140*Sin[c + d*x] + 190800*Sin[2*(c + d*x)] + 99500*Sin[3*(c + d*x)]/(393216*d*(3 + 5*Cos[c + d*x])^3)

Maple [A]

time = 0.10, size = 124, normalized size = 0.89

method	result
norman	$\frac{-\frac{295 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{512d} + \frac{265 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{768d} - \frac{745 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8192d}}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 4\right)^3} + \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{32768d} - \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{32768d}$
derivativedivides	$\frac{-\frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^3} + \frac{175}{4096 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} - \frac{745}{16384 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{32768} - \frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^3}}{d}$
default	$\frac{-\frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^3} + \frac{175}{4096 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} - \frac{745}{16384 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{32768} - \frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^3}}{d}$
risch	$\frac{i(20925 e^{5i(dx+c)} + 62775 e^{4i(dx+c)} + 111042 e^{3i(dx+c)} + 119310 e^{2i(dx+c)} + 68625 e^{i(dx+c)} + 24875)}{12288d(5 e^{2i(dx+c)} + 6 e^{i(dx+c)} + 5)^3} + \frac{279 \ln(e^{i(dx+c)} + 2)}{32768d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3+5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-125/6144/(tan(1/2*d*x+1/2*c)+2)^3+175/4096/(tan(1/2*d*x+1/2*c)+2)^2-745/16384/(tan(1/2*d*x+1/2*c)+2)-279/32768*ln(tan(1/2*d*x+1/2*c)+2)-125/6144/(tan(1/2*d*x+1/2*c)-2)^3-175/4096/(tan(1/2*d*x+1/2*c)-2)^2-745/16384/(tan(1/2*d*x+1/2*c)-2)+279/32768*ln(tan(1/2*d*x+1/2*c)-2))

Maxima [A]

time = 0.30, size = 174, normalized size = 1.24

$$\frac{20 \left(\frac{2832 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1696 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{447 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{\frac{48 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{12 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - 64} + 837 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2\right) - 837 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2\right)$$

98304 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/98304*(20*(2832*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 447*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) + 837*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 837*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d

Fricas [A]

time = 0.38, size = 170, normalized size = 1.21

$$\frac{837(125 \cos(dx+c)^3 + 225 \cos(dx+c)^2 + 135 \cos(dx+c) + 27) \log\left(\frac{1}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 837(125 \cos(dx+c)^3 + 225 \cos(dx+c)^2 + 135 \cos(dx+c) + 27) \log\left(\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) - 40(4975 \cos(dx+c)^2 + 4770 \cos(dx+c) + 1583) \sin(dx+c)}{196608(125 d \cos(dx+c)^3 + 225 d \cos(dx+c)^2 + 135 d \cos(dx+c) + 27 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\frac{-1/196608*(837*(125*\cos(d*x + c)^3 + 225*\cos(d*x + c)^2 + 135*\cos(d*x + c) + 27)*\log(3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - 837*(125*\cos(d*x + c)^3 + 225*\cos(d*x + c)^2 + 135*\cos(d*x + c) + 27)*\log(3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) - 40*(4975*\cos(d*x + c)^2 + 4770*\cos(d*x + c) + 1583)*\sin(d*x + c)}{(125*d*\cos(d*x + c)^3 + 225*d*\cos(d*x + c)^2 + 135*d*\cos(d*x + c) + 27*d)}$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(126) = 252.

time = 2.76, size = 813, normalized size = 5.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))**4,x)

[Out] Piecewise((x/(5*cos(2*atan(2)) + 3)**4, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*x + 2*atan(2))), (x/(5*cos(c) + 3)**4, Eq(d, 0)), (837*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 10044*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 40176*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 53568*log(tan(c/2 + d*x/2) - 2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 837*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 10044*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 40176*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 53568*log(tan(c/2 + d*x/2) + 2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 8940*tan(c/2 + d*x/2)**5/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 33920*tan(c/2 + d*x/2)**3/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 1179648*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 8940*tan(c/2 + d*x/2)**5/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 33920*tan(c/2 + d*x/2)**3/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 1179648*d*tan(c/2 + d*x/2)**2 - 6291456*d)

/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 56640*tan(c/2 + d*x/2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d), True))

Giac [A]

time = 0.50, size = 91, normalized size = 0.65

$$\frac{20 \left(447 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1696 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 2832 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 4 \right)^3} + 837 \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2 \right|\right) - 837 \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2 \right|\right)}{98304 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/98304*(20*(447*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 2832*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^3 + 837*log(abs(tan(1/2*d*x + 1/2*c) + 2)) - 837*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d

Mupad [B]

time = 1.97, size = 102, normalized size = 0.73

$$\frac{279 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{16384 d} - \frac{\frac{745 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8192} - \frac{265 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{768} + \frac{295 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{512}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 12 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 48 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 64 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*cos(c + d*x) + 3)^4,x)

[Out] - (279*atanh(tan(c/2 + (d*x)/2)/2))/(16384*d) - ((295*tan(c/2 + (d*x)/2))/512 - (265*tan(c/2 + (d*x)/2)^3)/768 + (745*tan(c/2 + (d*x)/2)^5)/8192)/(d*(48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 64))

$$3.38 \quad \int \frac{1}{3-5 \cos(c+dx)} dx$$

Optimal. Leaf size=63

$$\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

[Out] 1/4*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d-1/4*ln(cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 213}

$$\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(3 - 5*Cos[c + d*x])^(-1), x]

[Out] Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(4*d) - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(4*d)

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{3-5 \cos(c+dx)} dx = \frac{2\text{Subst}\left(\int \frac{1}{-2+8x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} = \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Mathematica [A]

time = 0.02, size = 63, normalized size = 1.00

$$\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 - 5*Cos[c + d*x])^(-1),x]``[Out] Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(4*d) - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(4*d)`**Maple [A]**

time = 0.05, size = 38, normalized size = 0.60

method	result	size
derivativedivides	$\frac{\frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{4} - \frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{4}}{d}$	38
default	$\frac{\frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{4} - \frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{4}}{d}$	38
norman	$\frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{4d} - \frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{4d}$	40
risch	$\frac{\ln\left(e^{i(dx+c)} - \frac{3}{5} - \frac{4i}{5}\right)}{4d} - \frac{\ln\left(e^{i(dx+c)} - \frac{3}{5} + \frac{4i}{5}\right)}{4d}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(3-5*cos(d*x+c)),x,method=_RETURNVERBOSE)``[Out] 1/d*(1/4*ln(2*tan(1/2*d*x+1/2*c)-1)-1/4*ln(2*tan(1/2*d*x+1/2*c)+1))`**Maxima [A]**

time = 0.28, size = 50, normalized size = 0.79

$$-\frac{\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} + 1\right) - \log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(3-5*cos(d*x+c)),x, algorithm="maxima")``[Out] -1/4*(log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d`**Fricas [A]**

time = 0.38, size = 46, normalized size = 0.73

$$-\frac{\log\left(-\frac{3}{2}\cos(dx+c) + 2\sin(dx+c) + \frac{5}{2}\right) - \log\left(-\frac{3}{2}\cos(dx+c) - 2\sin(dx+c) + \frac{5}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c)),x, algorithm="fricas")`

[Out] $-1/8*(\log(-3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - \log(-3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2))/d$

Sympy [A]

time = 0.36, size = 44, normalized size = 0.70

$$\begin{cases} \frac{\log\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 1\right)}{4d} - \frac{\log\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{3-5\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c)),x)`

[Out] `Piecewise((log(2*tan(c/2 + d*x/2) - 1)/(4*d) - log(2*tan(c/2 + d*x/2) + 1)/(4*d), Ne(d, 0)), (x/(3 - 5*cos(c)), True))`

Giac [A]

time = 0.44, size = 38, normalized size = 0.60

$$\frac{\log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c)),x, algorithm="giac")`

[Out] $-1/4*(\log(\text{abs}(2*\tan(1/2*d*x + 1/2*c) + 1)) - \log(\text{abs}(2*\tan(1/2*d*x + 1/2*c) - 1)))/d$

Mupad [B]

time = 0.33, size = 17, normalized size = 0.27

$$\frac{\operatorname{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(5*cos(c + d*x) - 3),x)`

[Out] $-\operatorname{atanh}(2*\tan(c/2 + (d*x)/2))/(2*d)$

$$3.39 \quad \int \frac{1}{(3-5 \cos(c+dx))^2} dx$$

Optimal. Leaf size=88

$$\frac{3 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{64d} + \frac{3 \log(\cos(\frac{1}{2}(c+dx)) + 2 \sin(\frac{1}{2}(c+dx)))}{64d} - \frac{5 \sin(c+dx)}{16d(3-5 \cos(c+dx))}$$

[Out] $-3/64*\ln(\cos(1/2*d*x+1/2*c)-2*\sin(1/2*d*x+1/2*c))/d+3/64*\ln(\cos(1/2*d*x+1/2*c)+2*\sin(1/2*d*x+1/2*c))/d-5/16*\sin(d*x+c)/d/(3-5*\cos(d*x+c))$

Rubi [A]

time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 12, 2738, 213}

$$-\frac{5 \sin(c+dx)}{16d(3-5 \cos(c+dx))} - \frac{3 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{64d} + \frac{3 \log(2 \sin(\frac{1}{2}(c+dx)) + \cos(\frac{1}{2}(c+dx)))}{64d}$$

Antiderivative was successfully verified.

[In] Int[(3 - 5*Cos[c + d*x])^(-2), x]

[Out] $(-3*\text{Log}[\text{Cos}[(c + d*x)/2] - 2*\text{Sin}[(c + d*x)/2]])/(64*d) + (3*\text{Log}[\text{Cos}[(c + d*x)/2] + 2*\text{Sin}[(c + d*x)/2]])/(64*d) - (5*\text{Sin}[c + d*x])/(16*d*(3 - 5*\text{Cos}[c + d*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3 - 5 \cos(c + dx))^2} dx &= -\frac{5 \sin(c + dx)}{16d(3 - 5 \cos(c + dx))} + \frac{1}{16} \int -\frac{3}{3 - 5 \cos(c + dx)} dx \\
&= -\frac{5 \sin(c + dx)}{16d(3 - 5 \cos(c + dx))} - \frac{3}{16} \int \frac{1}{3 - 5 \cos(c + dx)} dx \\
&= -\frac{5 \sin(c + dx)}{16d(3 - 5 \cos(c + dx))} - \frac{3 \text{Subst}\left(\int \frac{1}{-2+8x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{8d} \\
&= -\frac{3 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d} + \frac{3 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 139, normalized size = 1.58

$$\frac{9 \log(\cos(\frac{1}{2}(c + dx)) - 2 \sin(\frac{1}{2}(c + dx))) - 15 \cos(c + dx) (\log(\cos(\frac{1}{2}(c + dx)) - 2 \sin(\frac{1}{2}(c + dx))) - \log(\cos(\frac{1}{2}(c + dx)) + 2 \sin(\frac{1}{2}(c + dx)))) - 9 \log(\cos(\frac{1}{2}(c + dx)) + 2 \sin(\frac{1}{2}(c + dx))) + 20 \sin(c + dx)}{64d(-3 + 5 \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 5*Cos[c + d*x])^(-2),x]

[Out] (9*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 15*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]) - 9*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d*(-3 + 5*Cos[c + d*x]))

Maple [A]

time = 0.07, size = 72, normalized size = 0.82

method	result	size
norman	$-\frac{5 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16d\left(4\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)} - \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{64d} + \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{64d}$	71
derivativedivides	$-\frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{64} - \frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{64}$	72
default	$-\frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{64} - \frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{64}$	72

risch	$-\frac{i(3e^{i(dx+c)}-5)}{8d(5e^{2i(dx+c)}-6e^{i(dx+c)}+5)} - \frac{3\ln(e^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5})}{64d} + \frac{3\ln(e^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5})}{64d}$	85
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3-5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out] $1/d*(-5/64/(2*\tan(1/2*d*x+1/2*c)-1)-3/64*\ln(2*\tan(1/2*d*x+1/2*c)-1)-5/64/(2*\tan(1/2*d*x+1/2*c)+1)+3/64*\ln(2*\tan(1/2*d*x+1/2*c)+1))$

Maxima [A]

time = 0.30, size = 94, normalized size = 1.07

$$\frac{\frac{20 \sin(dx+c)}{\left(\frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)} - 3 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1\right) + 3 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c))^2,x, algorithm="maxima")`

[Out] $-1/64*(20*\sin(d*x + c)/((4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 - 1)*(\cos(d*x + c) + 1)) - 3*\log(2*\sin(d*x + c)/(\cos(d*x + c) + 1) + 1) + 3*\log(2*\sin(d*x + c)/(\cos(d*x + c) + 1) - 1))/d$

Fricas [A]

time = 0.37, size = 88, normalized size = 1.00

$$\frac{3(5 \cos(dx+c) - 3) \log\left(-\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 3(5 \cos(dx+c) - 3) \log\left(-\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) + 40 \sin(dx+c)}{128(5d \cos(dx+c) - 3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] $1/128*(3*(5*\cos(d*x + c) - 3)*\log(-3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - 3*(5*\cos(d*x + c) - 3)*\log(-3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) + 40*\sin(d*x + c))/(5*d*\cos(d*x + c) - 3*d)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(78) = 156.

time = 0.70, size = 240, normalized size = 2.73

$$\begin{cases} \frac{x}{(3-5\cos(2\operatorname{atan}(\frac{1}{2})))^2} & \text{for } c = -dx - 2\operatorname{atan}(\frac{1}{2}) \vee c = -dx + 2\operatorname{atan}(\frac{1}{2}) \\ \frac{x}{(3-5\cos(c))^2} & \text{for } d = 0 \\ -\frac{12\log(2\tan(\frac{c}{2}+\frac{dx}{2})-1)\tan^2(\frac{c}{2}+\frac{dx}{2})}{256d\tan^2(\frac{c}{2}+\frac{dx}{2})-64d} + \frac{3\log(2\tan(\frac{c}{2}+\frac{dx}{2})-1)}{256d\tan^2(\frac{c}{2}+\frac{dx}{2})-64d} + \frac{12\log(2\tan(\frac{c}{2}+\frac{dx}{2})+1)\tan^2(\frac{c}{2}+\frac{dx}{2})}{256d\tan^2(\frac{c}{2}+\frac{dx}{2})-64d} - \frac{3\log(2\tan(\frac{c}{2}+\frac{dx}{2})+1)}{256d\tan^2(\frac{c}{2}+\frac{dx}{2})-64d} - \frac{20\tan(\frac{c}{2}+\frac{dx}{2})}{256d\tan^2(\frac{c}{2}+\frac{dx}{2})-64d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c))**2,x)`

[Out] Piecewise((x/(3 - 5*cos(2*atan(1/2)))**2, Eq(c, -d*x - 2*atan(1/2)) | Eq(c, -d*x + 2*atan(1/2))), (x/(3 - 5*cos(c))**2, Eq(d, 0)), (-12*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 3*log(2*tan(c/2 + d*x/2) - 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 12*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) - 3*log(2*tan(c/2 + d*x/2) + 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) - 20*tan(c/2 + d*x/2)/(256*d*tan(c/2 + d*x/2)**2 - 64*d), True))

Giac [A]

time = 0.52, size = 68, normalized size = 0.77

$$\frac{20 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} - 3 \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) + 3 \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-5*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(4*tan(1/2*d*x + 1/2*c)^2 - 1) - 3*log(abs(2*tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d

Mupad [B]

time = 0.34, size = 47, normalized size = 0.53

$$\frac{3 \operatorname{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{32 d} - \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{64 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - \frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*cos(c + d*x) - 3)^2,x)

[Out] (3*atanh(2*tan(c/2 + (d*x)/2)))/(32*d) - (5*tan(c/2 + (d*x)/2))/(64*d*(tan(c/2 + (d*x)/2)^2 - 1/4))

$$3.40 \quad \int \frac{1}{(3-5 \cos(c+dx))^3} dx$$

Optimal. Leaf size=113

$$\frac{43 \log \left(\cos \left(\frac{1}{2}(c+dx) \right) - 2 \sin \left(\frac{1}{2}(c+dx) \right) \right)}{2048d} - \frac{43 \log \left(\cos \left(\frac{1}{2}(c+dx) \right) + 2 \sin \left(\frac{1}{2}(c+dx) \right) \right)}{2048d} - \frac{5 \sin(c+dx)}{32d(3-5 \cos(c+dx))}$$

[Out] 43/2048*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d-43/2048*ln(cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d-5/32*sin(d*x+c)/d/(3-5*cos(d*x+c))^2+45/512*2*sin(d*x+c)/d/(3-5*cos(d*x+c))

Rubi [A]

time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$,

Rules used = {2743, 2833, 12, 2738, 213}

$$\frac{45 \sin(c+dx)}{512d(3-5 \cos(c+dx))} - \frac{5 \sin(c+dx)}{32d(3-5 \cos(c+dx))^2} + \frac{43 \log \left(\cos \left(\frac{1}{2}(c+dx) \right) - 2 \sin \left(\frac{1}{2}(c+dx) \right) \right)}{2048d} - \frac{43 \log \left(2 \sin \left(\frac{1}{2}(c+dx) \right) + \cos \left(\frac{1}{2}(c+dx) \right) \right)}{2048d}$$

Antiderivative was successfully verified.

[In] Int[(3 - 5*Cos[c + d*x])^(-3), x]

[Out] (43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(2048*d) - (43*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(2048*d) - (5*Sin[c + d*x])/(32*d*(3 - 5*Cos[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist

`[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]`

Rule 2833

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]`

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(3 - 5 \cos(c + dx))^3} dx &= -\frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} + \frac{1}{32} \int \frac{-6 - 5 \cos(c + dx)}{(3 - 5 \cos(c + dx))^2} dx \\
 &= -\frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))} + \frac{1}{512} \int \frac{43}{3 - 5 \cos(c + dx)} dx \\
 &= -\frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))} + \frac{43}{512} \int \frac{1}{3 - 5 \cos(c + dx)} dx \\
 &= -\frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))} + \frac{43 \text{Subst}\left(\int \frac{1}{-2+8x^2} dx, x, \frac{1}{2}(c + dx)\right)}{256d} \\
 &= \frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} - \frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d}
 \end{aligned}$$

Mathematica [A]

time = 0.10, size = 211, normalized size = 1.87

$$\frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} - \frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} - \frac{5}{512d \left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} - \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{1024d \left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)} + \frac{5}{512d \left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} - \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{1024d \left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 - 5*Cos[c + d*x])^(-3), x]`

`[Out] (43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(2048*d) - (43*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(2048*d) - 5/(512*d*(Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2])^2) - (45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2])) + 5/(512*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2])^2) - (45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]))`

Maple [A]

time = 0.08, size = 106, normalized size = 0.94

method	result
norman	$\frac{-\frac{85 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 35 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512d} + \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2048d} - \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2048d}$
derivativedivides	$\frac{-\frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2048} + \frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}}{d}$
default	$\frac{-\frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2048} + \frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}}{d}$
risch	$\frac{i(215 e^{3i(dx+c)} - 387 e^{2i(dx+c)} + 325 e^{i(dx+c)} - 225)}{256d(5 e^{2i(dx+c)} - 6 e^{i(dx+c)} + 5)^2} + \frac{43 \ln\left(e^{i(dx+c)} - \frac{3}{5} - \frac{4i}{5}\right)}{2048d} - \frac{43 \ln\left(e^{i(dx+c)} - \frac{3}{5} + \frac{4i}{5}\right)}{2048d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3-5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] $1/d * (-25/2048 / (2 * \tan(1/2 * d * x + 1/2 * c) - 1)^2 + 35/2048 / (2 * \tan(1/2 * d * x + 1/2 * c) - 1) + 43/2048 * \ln(2 * \tan(1/2 * d * x + 1/2 * c) - 1) + 25/2048 / (2 * \tan(1/2 * d * x + 1/2 * c) + 1)^2 + 35/2048 / (2 * \tan(1/2 * d * x + 1/2 * c) + 1) - 43/2048 * \ln(2 * \tan(1/2 * d * x + 1/2 * c) + 1))$

Maxima [A]

time = 0.30, size = 137, normalized size = 1.21

$$\frac{20 \left(\frac{17 \sin(dx+c)}{\cos(dx+c)+1} - \frac{28 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) - 43 \log \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1 \right) + 43 \log \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1 \right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $1/2048 * (20 * (17 * \sin(d * x + c) / (\cos(d * x + c) + 1) - 28 * \sin(d * x + c)^3 / (\cos(d * x + c) + 1)^3) / (8 * \sin(d * x + c)^2 / (\cos(d * x + c) + 1)^2 - 16 * \sin(d * x + c)^4 / (\cos(d * x + c) + 1)^4 - 1) - 43 * \log(2 * \sin(d * x + c) / (\cos(d * x + c) + 1) + 1) + 43 * \log(2 * \sin(d * x + c) / (\cos(d * x + c) + 1) - 1)) / d$

Fricas [A]

time = 0.41, size = 129, normalized size = 1.14

$$\frac{-43(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9) \log\left(-\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 43(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9) \log\left(-\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) + 40(45 \cos(dx+c) - 11) \sin(dx+c)}{4096(25d \cos(dx+c)^2 - 30d \cos(dx+c) + 9d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3-5*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $-1/4096 * (43 * (25 * \cos(d * x + c)^2 - 30 * \cos(d * x + c) + 9) * \log(-3/2 * \cos(d * x + c) + 2 * \sin(d * x + c) + 5/2) - 43 * (25 * \cos(d * x + c)^2 - 30 * \cos(d * x + c) + 9) * \log$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(5*cos(c + d*x) - 3)^3,x)
```

```
[Out] - (43*atanh(2*tan(c/2 + (d*x)/2)))/(1024*d) - ((85*tan(c/2 + (d*x)/2))/8192  
- (35*tan(c/2 + (d*x)/2)^3)/2048)/(d*(tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*  
x)/2)^2/2 + 1/16))
```

3.41 $\int \frac{1}{(3-5 \cos(c+dx))^4} dx$

Optimal. Leaf size=138

$$\frac{279 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{32768d} + \frac{279 \log(\cos(\frac{1}{2}(c+dx)) + 2 \sin(\frac{1}{2}(c+dx)))}{32768d} - \frac{5 \sin(c)}{48d(3-5 \cos(c+dx))}$$

[Out] -279/32768*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+279/32768*ln(cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d-5/48*sin(d*x+c)/d/(3-5*cos(d*x+c))^3+25/512*sin(d*x+c)/d/(3-5*cos(d*x+c))^2-995/24576*sin(d*x+c)/d/(3-5*cos(d*x+c))

Rubi [A]

time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 213}

$$-\frac{995 \sin(c+dx)}{24576d(3-5 \cos(c+dx))} + \frac{25 \sin(c+dx)}{512d(3-5 \cos(c+dx))^2} - \frac{5 \sin(c+dx)}{48d(3-5 \cos(c+dx))^3} - \frac{279 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{32768d} + \frac{279 \log(2 \sin(\frac{1}{2}(c+dx)) + \cos(\frac{1}{2}(c+dx)))}{32768d}$$

Antiderivative was successfully verified.

[In] Int[(3 - 5*Cos[c + d*x])^(-4), x]

[Out] (-279*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(32768*d) + (279*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(32768*d) - (5*Sin[c + d*x])/(48*d*(3 - 5*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x])^2) - (995*Sin[c + d*x])/(24576*d*(3 - 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3 - 5 \cos(c + dx))^4} dx &= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{1}{48} \int \frac{-9 - 10 \cos(c + dx)}{(3 - 5 \cos(c + dx))^3} dx \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} + \frac{\int \frac{154 + 75 \cos(c + dx)}{(3 - 5 \cos(c + dx))^2} dx}{1536} \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} - \frac{995 \sin(c + dx)}{24576d(3 - 5 \cos(c + dx))} \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} - \frac{995 \sin(c + dx)}{24576d(3 - 5 \cos(c + dx))} \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} - \frac{995 \sin(c + dx)}{24576d(3 - 5 \cos(c + dx))} \\
&= -\frac{279 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{32768d} + \frac{279 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{32768d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 288 vs. 2(138) = 276.

time = 0.16, size = 288, normalized size = 2.09

877648*log(1/2 + d*x) - 288*(3 + d*x) - 104625*cos(3*(c + d*x)) - 765855*cos(c + d*x)*(Log[Cos((c + d*x)/2) - 2*Sin((c + d*x)/2)] - 2*Sin((c + d*x)/2)) - 995*Sin(c + d*x) - 24576*d*(3 - 5*cos(c + d*x)) - 279*log(cos(1/2*(c + d*x)) - 2*Sin(1/2*(c + d*x))) + 279*log(cos(1/2*(c + d*x)) + 2*Sin(1/2*(c + d*x))) - 1536*Int[154 + 75*cos(c + dx)/(3 - 5*cos(c + dx))^2, dx]

Antiderivative was successfully verified.

[In] Integrate[(3 - 5*Cos[c + d*x])^(-4), x]

[Out] (467046*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 765855*Cos[c + d*x]*(Log[Cos

$$\begin{aligned} & [(c + d*x)/2] - 2*\text{Sin}[(c + d*x)/2]] - \text{Log}[\text{Cos}[(c + d*x)/2] + 2*\text{Sin}[(c + d*x) \\ &)/2]]) + 376650*\text{Cos}[2*(c + d*x)]*(\text{Log}[\text{Cos}[(c + d*x)/2] - 2*\text{Sin}[(c + d*x)/2] \\ &] - \text{Log}[\text{Cos}[(c + d*x)/2] + 2*\text{Sin}[(c + d*x)/2]]) - 467046*\text{Log}[\text{Cos}[(c + d*x)/ \\ & 2] + 2*\text{Sin}[(c + d*x)/2]] + 104625*\text{Cos}[3*(c + d*x)]*\text{Log}[\text{Cos}[(c + d*x)/2] + 2 \\ & * \text{Sin}[(c + d*x)/2]] + 226140*\text{Sin}[c + d*x] - 190800*\text{Sin}[2*(c + d*x)] + 99500* \\ & \text{Sin}[3*(c + d*x)]/(393216*d*(-3 + 5*\text{Cos}[c + d*x])^3) \end{aligned}$$

Maple [A]

time = 0.10, size = 140, normalized size = 1.01

method	result
norman	$-\frac{745 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{265 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{768d} - \frac{295 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512d}}{\left(4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)^3} - \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{32768d} + \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{32768d}$
risch	$-\frac{i(20925 e^{5i(dx+c)} - 62775 e^{4i(dx+c)} + 111042 e^{3i(dx+c)} - 119310 e^{2i(dx+c)} + 68625 e^{i(dx+c)} - 24875)}{12288d(5 e^{2i(dx+c)} - 6 e^{i(dx+c)} + 5)^3} - \frac{279 \ln(e^{i(dx+c)} - 1)}{32768d}$
derivativedivides	$-\frac{\frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} + \frac{25}{8192 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{295}{32768 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{32768}}{d} - \frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{25}{8192 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{295}{32768 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{32768}$
default	$-\frac{\frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} + \frac{25}{8192 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{295}{32768 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{32768}}{d} - \frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{25}{8192 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{295}{32768 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{32768}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3-5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-125/49152/(2*tan(1/2*d*x+1/2*c)-1)^3+25/8192/(2*tan(1/2*d*x+1/2*c)-1)^2-295/32768/(2*tan(1/2*d*x+1/2*c)-1)-279/32768*ln(2*tan(1/2*d*x+1/2*c)-1)-125/49152/(2*tan(1/2*d*x+1/2*c)+1)^3-25/8192/(2*tan(1/2*d*x+1/2*c)+1)^2-295/32768/(2*tan(1/2*d*x+1/2*c)+1)+279/32768*ln(2*tan(1/2*d*x+1/2*c)+1))

Maxima [A]

time = 0.28, size = 177, normalized size = 1.28

$$-\frac{20 \left(\frac{447 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1696 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2832 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) - 837 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1\right) + 837 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{\frac{12 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{48 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{64 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - 1} 98304 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-5*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/98304*(20*(447*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2832*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(12*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 48*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 64*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1) - 837*log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) + 837*log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d

Fricas [A]

time = 0.37, size = 170, normalized size = 1.23

$$\frac{837(125 \cos(dx+c)^3 - 225 \cos(dx+c)^2 + 135 \cos(dx+c) - 27) \log\left(-\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 837(125 \cos(dx+c)^3 - 225 \cos(dx+c)^2 + 135 \cos(dx+c) - 27) \log\left(-\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) + 40(4975 \cos(dx+c)^2 - 4770 \cos(dx+c) + 1583) \sin(dx+c)}{196608(125 d \cos(dx+c)^3 - 225 d \cos(dx+c)^2 + 135 d \cos(dx+c) - 27 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-5*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/196608*(837*(125*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 135*cos(d*x + c) - 27)*log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - 837*(125*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 135*cos(d*x + c) - 27)*log(-3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 40*(4975*cos(d*x + c)^2 - 4770*cos(d*x + c) + 1583)*sin(d*x + c))/(125*d*cos(d*x + c)^3 - 225*d*cos(d*x + c)^2 + 135*d*cos(d*x + c) - 27*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(126) = 252.

time = 2.74, size = 831, normalized size = 6.02

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-5*cos(d*x+c))**4,x)

[Out] Piecewise((x/(3 - 5*cos(2*atan(1/2)))**4, Eq(c, -d*x - 2*atan(1/2)) | Eq(c, -d*x + 2*atan(1/2))), (x/(3 - 5*cos(c))**4, Eq(d, 0)), (-53568*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**6/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 40176*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**4/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) - 10044*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 837*log(2*tan(c/2 + d*x/2) - 1)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 53568*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) - 40176*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 10044*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) - 837*log(2*tan(c/2 + d*x/2) + 1)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) - 56640*tan(c/2 + d*x/2)**5/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 33920*tan(c/2 + d*x/2)**3/(6291456*d*tan(c/2 + d*x/2)**6

```
- 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d)
- 8940*tan(c/2 + d*x/2)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2
+ d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d), True))
```

Giac [A]

time = 0.43, size = 97, normalized size = 0.70

$$\frac{20 \left(2832 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1696 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 447 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right) - 837 \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) + 837 \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{\frac{(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1)^3}{98304 d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(20*(2832*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 4
47*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 - 1)^3 - 837*log(abs(2*t
an(1/2*d*x + 1/2*c) + 1)) + 837*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B]

time = 2.01, size = 102, normalized size = 0.74

$$\frac{279 \operatorname{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{16384 d} - \frac{\frac{295 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{32768} - \frac{265 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{49152} + \frac{745 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{524288}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{4} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{16} - \frac{1}{64} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cos(c + d*x) - 3)^4,x)
```

```
[Out] (279*atanh(2*tan(c/2 + (d*x)/2)))/(16384*d) - ((745*tan(c/2 + (d*x)/2))/524
288 - (265*tan(c/2 + (d*x)/2)^3)/49152 + (295*tan(c/2 + (d*x)/2)^5)/32768)/
(d*((3*tan(c/2 + (d*x)/2)^2)/16 - (3*tan(c/2 + (d*x)/2)^4)/4 + tan(c/2 + (d
*x)/2)^6 - 1/64))
```

$$3.42 \quad \int \frac{1}{-3+5 \cos(c+dx)} dx$$

Optimal. Leaf size=63

$$-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

[Out] $-1/4*\ln(\cos(1/2*d*x+1/2*c)-2*\sin(1/2*d*x+1/2*c))/d+1/4*\ln(\cos(1/2*d*x+1/2*c)+2*\sin(1/2*d*x+1/2*c))/d$

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 212}

$$\frac{\log\left(2\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-3 + 5*\text{Cos}[c + d*x])^{-1}, x]$

[Out] $-1/4*\text{Log}[\text{Cos}[(c + d*x)/2] - 2*\text{Sin}[(c + d*x)/2]]/d + \text{Log}[\text{Cos}[(c + d*x)/2] + 2*\text{Sin}[(c + d*x)/2]]/(4*d)$

Rule 212

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2738

$\text{Int}[(a_ + (b_)*\sin[\text{Pi}/2 + (c_ + (d_)*(x_))])^{-1}, x_Symbol] \rightarrow \text{With}\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[2*(e/d), \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{-3+5 \cos(c+dx)} dx &= \frac{2\text{Subst}\left(\int \frac{1}{2-8x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} \\ &= -\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 63, normalized size = 1.00

$$-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

`[In] Integrate[(-3 + 5*Cos[c + d*x])^(-1),x]``[Out] -1/4*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/d + Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(4*d)`**Maple [A]**

time = 0.05, size = 38, normalized size = 0.60

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{4} + \frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{4}}{d}$	38
default	$\frac{-\frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{4} + \frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{4}}{d}$	38
norman	$-\frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{4d} + \frac{\ln\left(2\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{4d}$	40
risch	$\frac{\ln\left(e^{i(dx+c)} - \frac{3}{5} + \frac{4i}{5}\right)}{4d} - \frac{\ln\left(e^{i(dx+c)} - \frac{3}{5} - \frac{4i}{5}\right)}{4d}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-3+5*cos(d*x+c)),x,method=_RETURNVERBOSE)``[Out] 1/d*(-1/4*ln(2*tan(1/2*d*x+1/2*c)-1)+1/4*ln(2*tan(1/2*d*x+1/2*c)+1))`**Maxima [A]**

time = 0.28, size = 50, normalized size = 0.79

$$\frac{\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} + 1\right) - \log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(-3+5*cos(d*x+c)),x, algorithm="maxima")``[Out] 1/4*(log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d`**Fricas [A]**

time = 0.37, size = 46, normalized size = 0.73

$$\frac{\log\left(-\frac{3}{2}\cos(dx+c) + 2\sin(dx+c) + \frac{5}{2}\right) - \log\left(-\frac{3}{2}\cos(dx+c) - 2\sin(dx+c) + \frac{5}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c)),x, algorithm="fricas")`

[Out] $\frac{1}{8} * (\log(-3/2 * \cos(d*x + c) + 2 * \sin(d*x + c) + 5/2) - \log(-3/2 * \cos(d*x + c) - 2 * \sin(d*x + c) + 5/2)) / d$

Sympy [A]

time = 0.33, size = 44, normalized size = 0.70

$$\begin{cases} -\frac{\log\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 1\right)}{4d} + \frac{\log\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{5 \cos(c) - 3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c)),x)`

[Out] `Piecewise((-log(2*tan(c/2 + d*x/2) - 1)/(4*d) + log(2*tan(c/2 + d*x/2) + 1)/(4*d), Ne(d, 0)), (x/(5*cos(c) - 3), True))`

Giac [A]

time = 0.49, size = 38, normalized size = 0.60

$$\frac{\log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c)),x, algorithm="giac")`

[Out] $\frac{1}{4} * (\log(\text{abs}(2 * \tan(1/2 * d * x + 1/2 * c) + 1)) - \log(\text{abs}(2 * \tan(1/2 * d * x + 1/2 * c) - 1))) / d$

Mupad [B]

time = 0.26, size = 17, normalized size = 0.27

$$\frac{\text{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*cos(c + d*x) - 3),x)`

[Out] `atanh(2*tan(c/2 + (d*x)/2))/(2*d)`

3.43 $\int \frac{1}{(-3+5 \cos(c+dx))^2} dx$

Optimal. Leaf size=88

$$-\frac{3 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{64d} + \frac{3 \log(\cos(\frac{1}{2}(c+dx)) + 2 \sin(\frac{1}{2}(c+dx)))}{64d} - \frac{5 \sin(c+dx)}{16d(3-5 \cos(c+dx))}$$

[Out] $-3/64*\ln(\cos(1/2*d*x+1/2*c)-2*\sin(1/2*d*x+1/2*c))/d+3/64*\ln(\cos(1/2*d*x+1/2*c)+2*\sin(1/2*d*x+1/2*c))/d-5/16*\sin(d*x+c)/d/(3-5*\cos(d*x+c))$

Rubi [A]

time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {2743, 12, 2738, 212}

$$-\frac{5 \sin(c+dx)}{16d(3-5 \cos(c+dx))} - \frac{3 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{64d} + \frac{3 \log(2 \sin(\frac{1}{2}(c+dx)) + \cos(\frac{1}{2}(c+dx)))}{64d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-3 + 5*\text{Cos}[c + d*x])^{(-2)}, x]$

[Out] $(-3*\text{Log}[\text{Cos}[(c + d*x)/2] - 2*\text{Sin}[(c + d*x)/2]])/(64*d) + (3*\text{Log}[\text{Cos}[(c + d*x)/2] + 2*\text{Sin}[(c + d*x)/2]])/(64*d) - (5*\text{Sin}[c + d*x])/(16*d*(3 - 5*\text{Cos}[c + d*x]))$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 212

$\text{Int}[(a_)+(b_.)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2738

$\text{Int}[(a_)+(b_.)*\sin[\text{Pi}/2 + (c_.)+(d_.)*(x_)])^{(-1)}, x_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[2*(e/d), \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2743

$\text{Int}[(a_)+(b_.)*\sin[(c_.)+(d_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((a + b*\text{Sin}[c + d*x])^{(n+1)})/(d*(n+1)*(a^2 - b^2)), x] + \text{Dist}$

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-3 + 5 \cos(c + dx))^2} dx &= -\frac{5 \sin(c + dx)}{16d(3 - 5 \cos(c + dx))} + \frac{1}{16} \int \frac{3}{-3 + 5 \cos(c + dx)} dx \\ &= -\frac{5 \sin(c + dx)}{16d(3 - 5 \cos(c + dx))} + \frac{3}{16} \int \frac{1}{-3 + 5 \cos(c + dx)} dx \\ &= -\frac{5 \sin(c + dx)}{16d(3 - 5 \cos(c + dx))} + \frac{3 \text{Subst}\left(\int \frac{1}{2-8x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{8d} \\ &= -\frac{3 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d} + \frac{3 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 139, normalized size = 1.58

$$\frac{9 \log(\cos(\frac{1}{2}(c + dx)) - 2 \sin(\frac{1}{2}(c + dx))) - 15 \cos(c + dx) (\log(\cos(\frac{1}{2}(c + dx)) - 2 \sin(\frac{1}{2}(c + dx))) - \log(\cos(\frac{1}{2}(c + dx)) + 2 \sin(\frac{1}{2}(c + dx)))) - 9 \log(\cos(\frac{1}{2}(c + dx)) + 2 \sin(\frac{1}{2}(c + dx))) + 20 \sin(c + dx)}{64d(-3 + 5 \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 5*Cos[c + d*x])^(-2), x]

[Out] (9*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 15*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]) - 9*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d*(-3 + 5*Cos[c + d*x]))

Maple [A]

time = 0.06, size = 72, normalized size = 0.82

method	result	size
norman	$-\frac{5 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16d\left(4\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)} - \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{64d} + \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{64d}$	71
derivativedivides	$-\frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{64} - \frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{64}$	72
default	$-\frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{64} - \frac{5}{64\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{3 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{64}$	72

risch	$-\frac{i(3e^{i(dx+c)}-5)}{8d(5e^{2i(dx+c)}-6e^{i(dx+c)}+5)} - \frac{3\ln(e^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5})}{64d} + \frac{3\ln(e^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5})}{64d}$	85
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3+5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out] `1/d*(-5/64/(2*tan(1/2*d*x+1/2*c)-1)-3/64*ln(2*tan(1/2*d*x+1/2*c)-1)-5/64/(2*tan(1/2*d*x+1/2*c)+1)+3/64*ln(2*tan(1/2*d*x+1/2*c)+1))`

Maxima [A]

time = 0.27, size = 94, normalized size = 1.07

$$\frac{\frac{20 \sin(dx+c)}{\left(\frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)(\cos(dx+c)+1)} - 3 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1\right) + 3 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c))^2,x, algorithm="maxima")`

[Out] `-1/64*(20*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)) - 3*log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) + 3*log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d`

Fricas [A]

time = 0.38, size = 88, normalized size = 1.00

$$\frac{3(5 \cos(dx+c) - 3) \log\left(-\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 3(5 \cos(dx+c) - 3) \log\left(-\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) + 40 \sin(dx+c)}{128(5d \cos(dx+c) - 3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] `1/128*(3*(5*cos(d*x + c) - 3)*log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - 3*(5*cos(d*x + c) - 3)*log(-3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 40*sin(d*x + c))/(5*d*cos(d*x + c) - 3*d)`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. $2(78) = 156$.

time = 0.69, size = 240, normalized size = 2.73

$$\begin{cases} \frac{\frac{x}{(-3+5 \cos(2 \operatorname{atan}(\frac{1}{2})))^2}}{(5 \cos(c)-3)^2} & \text{for } c = -dx - 2 \operatorname{atan}(\frac{1}{2}) \vee c = -dx + 2 \operatorname{atan}(\frac{1}{2}) \\ \frac{x}{(5 \cos(c)-3)^2} & \text{for } d = 0 \\ -\frac{12 \log\left(2 \tan\left(\frac{x}{2} + \frac{dx}{2}\right) - 1\right) \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right)}{256d \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right) - 64d} + \frac{3 \log\left(2 \tan\left(\frac{x}{2} + \frac{dx}{2}\right) - 1\right)}{256d \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right) - 64d} + \frac{12 \log\left(2 \tan\left(\frac{x}{2} + \frac{dx}{2}\right) + 1\right) \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right)}{256d \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right) - 64d} - \frac{3 \log\left(2 \tan\left(\frac{x}{2} + \frac{dx}{2}\right) + 1\right)}{256d \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right) - 64d} - \frac{20 \tan\left(\frac{x}{2} + \frac{dx}{2}\right)}{256d \tan^2\left(\frac{x}{2} + \frac{dx}{2}\right) - 64d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c))*2,x)`

[Out] Piecewise((x/(-3 + 5*cos(2*atan(1/2)))**2, Eq(c, -d*x - 2*atan(1/2)) | Eq(c, -d*x + 2*atan(1/2))), (x/(5*cos(c) - 3)**2, Eq(d, 0)), (-12*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 3*log(2*tan(c/2 + d*x/2) - 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 12*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) - 3*log(2*tan(c/2 + d*x/2) + 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) - 20*tan(c/2 + d*x/2)/(256*d*tan(c/2 + d*x/2)**2 - 64*d), True))

Giac [A]

time = 0.45, size = 68, normalized size = 0.77

$$\frac{20 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} - 3 \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) + 3 \log\left(\left|2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+5*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(4*tan(1/2*d*x + 1/2*c)^2 - 1) - 3*log(abs(2*tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d

Mupad [B]

time = 0.00, size = 47, normalized size = 0.53

$$\frac{3 \operatorname{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{32 d} - \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{64 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - \frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*cos(c + d*x) - 3)^2,x)

[Out] (3*atanh(2*tan(c/2 + (d*x)/2)))/(32*d) - (5*tan(c/2 + (d*x)/2))/(64*d*(tan(c/2 + (d*x)/2)^2 - 1/4))

3.44 $\int \frac{1}{(-3+5 \cos(c+dx))^3} dx$

Optimal. Leaf size=113

$$-\frac{43 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2 \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2 \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5 \sin(c+dx)}{32d(3-5 \cos(c+dx))}$$

[Out] -43/2048*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+43/2048*ln(cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d+5/32*sin(d*x+c)/d/(3-5*cos(d*x+c))^2-45/512*sin(d*x+c)/d/(3-5*cos(d*x+c))

Rubi [A]

time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$,

Rules used = {2743, 2833, 12, 2738, 212}

$$-\frac{45 \sin(c+dx)}{512d(3-5 \cos(c+dx))} + \frac{5 \sin(c+dx)}{32d(3-5 \cos(c+dx))^2} - \frac{43 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2 \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43 \log\left(2 \sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

Antiderivative was successfully verified.

[In] Int[(-3 + 5*Cos[c + d*x])^(-3), x]

[Out] (-43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(2048*d) + (43*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(2048*d) + (5*Sin[c + d*x])/(32*d*(3 - 5*Cos[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-3 + 5 \cos(c + dx))^3} dx &= \frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} + \frac{1}{32} \int \frac{6 + 5 \cos(c + dx)}{(-3 + 5 \cos(c + dx))^2} dx \\ &= \frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))} + \frac{1}{512} \int \frac{43}{-3 + 5 \cos(c + dx)} dx \\ &= \frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))} + \frac{43}{512} \int \frac{1}{-3 + 5 \cos(c + dx)} dx \\ &= \frac{5 \sin(c + dx)}{32d(3 - 5 \cos(c + dx))^2} - \frac{45 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))} + \frac{43 \text{Subst}\left(\int \frac{1}{2-8x^2} dx, x, \frac{1}{2}(c + dx)\right)}{256d} \\ &= -\frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} + \frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} \end{aligned}$$

Mathematica [A]

time = 0.09, size = 211, normalized size = 1.87

$$-\frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} + \frac{43 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} + \frac{5}{512d\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{1024d\left(\cos\left(\frac{1}{2}(c + dx)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)} - \frac{5}{512d\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{1024d\left(\cos\left(\frac{1}{2}(c + dx)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*Cos[c + d*x])^(-3), x]
```

```
[Out] (-43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(2048*d) + (43*Log[Cos[(c
+ d*x)/2] + 2*Sin[(c + d*x)/2]]/(2048*d) + 5/(512*d*(Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2])^2) + (45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2])) - 5/(512*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2])^2) +
(45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]))
```

Maple [A]

time = 0.08, size = 106, normalized size = 0.94

method	result
norman	$\frac{85 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 35 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512d} - \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2048d} + \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2048d}$
derivativedivides	$\frac{\frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2048} - \frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}}{d}$
default	$\frac{\frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{43 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2048} - \frac{25}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{35}{2048 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}}{d}$
risch	$-\frac{i(215 e^{3i(dx+c)} - 387 e^{2i(dx+c)} + 325 e^{i(dx+c)} - 225)}{256d(5 e^{2i(dx+c)} - 6 e^{i(dx+c)} + 5)^2} - \frac{43 \ln\left(e^{i(dx+c)} - \frac{3}{5} - \frac{4i}{5}\right)}{2048d} + \frac{43 \ln\left(e^{i(dx+c)} - \frac{3}{5} + \frac{4i}{5}\right)}{2048d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3+5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{d} \left(\frac{25}{2048} \left(2 \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^{-2} - \frac{35}{2048} \left(2 \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right) - \frac{43}{2048} \ln\left(2 \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right) - \frac{25}{2048} \left(2 \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + 1 \right)^{-2} - \frac{35}{2048} \left(2 \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + 1 \right) + \frac{43}{2048} \ln\left(2 \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + 1 \right) \right)$

Maxima [A]

time = 0.27, size = 137, normalized size = 1.21

$$\frac{20 \left(\frac{17 \sin(dx+c)}{\cos(dx+c)+1} - \frac{28 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) - 43 \log \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1 \right) + 43 \log \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1 \right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $-\frac{1}{2048} \left(20 \left(\frac{17 \sin(dx+c)}{\cos(dx+c)+1} - \frac{28 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) - 43 \log \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1 \right) + 43 \log \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1 \right) \right) / d$

Fricas [A]

time = 0.39, size = 129, normalized size = 1.14

$$\frac{43(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9) \log\left(-\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 43(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9) \log\left(-\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) + 40(45 \cos(dx+c) - 11) \sin(dx+c)}{4096(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $\frac{1}{4096} \left(43 \left(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9 \right) \log\left(-\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 43 \left(25 \cos(dx+c)^2 - 30 \cos(dx+c) + 9 \right) \log\left(-\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) + 40(45 \cos(dx+c) - 11) \sin(dx+c) \right) / d$

$-3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) + 40*(45*\cos(d*x + c) - 11)*\sin(d*x + c))/(25*d*\cos(d*x + c)^2 - 30*d*\cos(d*x + c) + 9*d)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(102) = 204$.

time = 1.27, size = 490, normalized size = 4.34

$$\frac{\int \frac{1}{-3+5\cos(dx+c)} dx}{\int \frac{1}{-3+5\cos(dx+c)} dx} = \frac{\int \frac{1}{-3+5\cos(dx+c)} dx}{\int \frac{1}{-3+5\cos(dx+c)} dx} \quad \text{for } c = -dx - 2\operatorname{atan}\left(\frac{1}{2}\right) \vee c = -dx + 2\operatorname{atan}\left(\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+5*cos(d*x+c))**3,x)

[Out] Piecewise((x/(-3 + 5*cos(2*atan(1/2)))**3, Eq(c, -d*x - 2*atan(1/2)) | Eq(c, -d*x + 2*atan(1/2))), (x/(5*cos(c) - 3)**3, Eq(d, 0)), (-688*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**4/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) + 344*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) - 43*log(2*tan(c/2 + d*x/2) - 1)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) + 688*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) - 344*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) + 43*log(2*tan(c/2 + d*x/2) + 1)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) - 560*tan(c/2 + d*x/2)**3/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) + 340*tan(c/2 + d*x/2)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d), True))

Giac [A]

time = 0.42, size = 84, normalized size = 0.74

$$\frac{20 \left(28 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 17 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1 \right)^2} - 43 \log\left(\left| 2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right|\right) + 43 \log\left(\left| 2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right|\right)$$

2048 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+5*cos(d*x+c))^3,x, algorithm="giac")

[Out] $-1/2048*(20*(28*\tan(1/2*d*x + 1/2*c)^3 - 17*\tan(1/2*d*x + 1/2*c))/(4*\tan(1/2*d*x + 1/2*c)^2 - 1)^2 - 43*\log(\operatorname{abs}(2*\tan(1/2*d*x + 1/2*c) + 1)) + 43*\log(\operatorname{abs}(2*\tan(1/2*d*x + 1/2*c) - 1)))/d$

Mupad [B]

time = 0.65, size = 75, normalized size = 0.66

$$\frac{43 \operatorname{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{1024 d} + \frac{\frac{85 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8192} - \frac{35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{2048}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{2} + \frac{1}{16} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cos(c + d*x) - 3)^3,x)
```

```
[Out] (43*atanh(2*tan(c/2 + (d*x)/2)))/(1024*d) + ((85*tan(c/2 + (d*x)/2))/8192 -  
(35*tan(c/2 + (d*x)/2)^3)/2048)/(d*(tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)  
/2)^2/2 + 1/16))
```

$$3.45 \quad \int \frac{1}{(-3+5 \cos(c+dx))^4} dx$$

Optimal. Leaf size=138

$$-\frac{279 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{32768d} + \frac{279 \log(\cos(\frac{1}{2}(c+dx)) + 2 \sin(\frac{1}{2}(c+dx)))}{32768d} - \frac{5 \sin(c)}{48d(3-5 \cos(c))}$$

[Out] -279/32768*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+279/32768*ln(cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d-5/48*sin(d*x+c)/d/(3-5*cos(d*x+c))^3+25/512*sin(d*x+c)/d/(3-5*cos(d*x+c))^2-995/24576*sin(d*x+c)/d/(3-5*cos(d*x+c))

Rubi [A]

time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 212}

$$-\frac{995 \sin(c+dx)}{24576d(3-5 \cos(c+dx))} + \frac{25 \sin(c+dx)}{512d(3-5 \cos(c+dx))^2} - \frac{5 \sin(c+dx)}{48d(3-5 \cos(c+dx))^3} - \frac{279 \log(\cos(\frac{1}{2}(c+dx)) - 2 \sin(\frac{1}{2}(c+dx)))}{32768d} + \frac{279 \log(2 \sin(\frac{1}{2}(c+dx)) + \cos(\frac{1}{2}(c+dx)))}{32768d}$$

Antiderivative was successfully verified.

[In] Int[(-3 + 5*Cos[c + d*x])^(-4), x]

[Out] (-279*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(32768*d) + (279*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]/(32768*d) - (5*Sin[c + d*x])/(48*d*(3 - 5*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x])^2) - (995*Sin[c + d*x])/(24576*d*(3 - 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-3 + 5 \cos(c + dx))^4} dx &= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{1}{48} \int \frac{9 + 10 \cos(c + dx)}{(-3 + 5 \cos(c + dx))^3} dx \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} + \frac{\int \frac{154 + 75 \cos(c + dx)}{(-3 + 5 \cos(c + dx))^2} dx}{1536} \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} - \frac{995 \sin(c + dx)}{24576d(3 - 5 \cos(c + dx))} \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} - \frac{995 \sin(c + dx)}{24576d(3 - 5 \cos(c + dx))} \\
&= -\frac{5 \sin(c + dx)}{48d(3 - 5 \cos(c + dx))^3} + \frac{25 \sin(c + dx)}{512d(3 - 5 \cos(c + dx))^2} - \frac{995 \sin(c + dx)}{24576d(3 - 5 \cos(c + dx))} \\
&= -\frac{279 \log(\cos(\frac{1}{2}(c + dx)) - 2 \sin(\frac{1}{2}(c + dx)))}{32768d} + \frac{279 \log(\cos(\frac{1}{2}(c + dx)) + 2 \sin(\frac{1}{2}(c + dx)))}{32768d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 288 vs. 2(138) = 276.

time = 0.02, size = 288, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*Cos[c + d*x])^(-4), x]
```

```
[Out] (467046*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)]
)*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 765855*Cos[c + d*x]*(Log[Cos
```


$$\left[\left(\frac{c + dx}{2} \right) - 2 \sin\left(\frac{c + dx}{2}\right) \right] - \log\left[\cos\left(\frac{c + dx}{2}\right) + 2 \sin\left(\frac{c + dx}{2}\right)\right] + 376650 \cos[2(c + dx)] \cdot \left(\log\left[\cos\left(\frac{c + dx}{2}\right) - 2 \sin\left(\frac{c + dx}{2}\right)\right] - \log\left[\cos\left(\frac{c + dx}{2}\right) + 2 \sin\left(\frac{c + dx}{2}\right)\right] \right) - 467046 \log\left[\cos\left(\frac{c + dx}{2}\right) + 2 \sin\left(\frac{c + dx}{2}\right)\right] + 104625 \cos[3(c + dx)] \cdot \left(\log\left[\cos\left(\frac{c + dx}{2}\right) + 2 \sin\left(\frac{c + dx}{2}\right)\right] + 226140 \sin[c + dx] - 190800 \sin[2(c + dx)] + 99500 \sin[3(c + dx)] \right) / (393216 d (-3 + 5 \cos[c + dx])^3)$$

Maple [A]

time = 0.09, size = 140, normalized size = 1.01

method	result
norman	$\frac{-\frac{745 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8192d} + \frac{265 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{768d} - \frac{295 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{512d}}{\left(4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)^3} - \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{32768d} + \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{32768d}$
risch	$\frac{i(20925 e^{5i(dx+c)} - 62775 e^{4i(dx+c)} + 111042 e^{3i(dx+c)} - 119310 e^{2i(dx+c)} + 68625 e^{i(dx+c)} - 24875)}{12288d(5 e^{2i(dx+c)} - 6 e^{i(dx+c)} + 5)^3} - \frac{279 \ln(e^{i(dx+c)} - 1)}{32768d}$
derivativedivides	$\frac{-\frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} + \frac{25}{8192 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{295}{32768 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{32768}}{d} - \frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3}$
default	$\frac{-\frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} + \frac{25}{8192 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{295}{32768 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{279 \ln\left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{32768}}{d} - \frac{125}{49152 \left(2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3+5*cos(dx+c))^4,x,method=_RETURNVERBOSE)`

[Out] $1/d \cdot (-125/49152 / (2 \tan(1/2 dx + 1/2 c) - 1)^3 + 25/8192 / (2 \tan(1/2 dx + 1/2 c) - 1)^2 - 295/32768 / (2 \tan(1/2 dx + 1/2 c) - 1) - 279/32768 \ln(2 \tan(1/2 dx + 1/2 c) - 1) - 125/49152 / (2 \tan(1/2 dx + 1/2 c) + 1)^3 - 25/8192 / (2 \tan(1/2 dx + 1/2 c) + 1)^2 - 295/32768 / (2 \tan(1/2 dx + 1/2 c) + 1) + 279/32768 \ln(2 \tan(1/2 dx + 1/2 c) + 1))$

Maxima [A]

time = 0.29, size = 177, normalized size = 1.28

$$\frac{20 \left(\frac{447 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1696 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2832 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{\frac{12 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{48 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{64 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - 1} - \frac{837 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1\right) + 837 \log\left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{98304 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+5*cos(dx+c))^4,x, algorithm="maxima")`

[Out] $-1/98304 \cdot (20 \cdot (447 \sin(dx+c) / (\cos(dx+c)+1) - 1696 \sin(dx+c)^3 / (\cos(dx+c)+1)^3 + 2832 \sin(dx+c)^5 / (\cos(dx+c)+1)^5) / (12 \sin(dx+c)^2 / (\cos(dx+c)+1)^2 - 48 \sin(dx+c)^4 / (\cos(dx+c)+1)^4 + 64 \sin(dx+c)^6 / (\cos(dx+c)+1)^6 - 1) - 837 \cdot \log(2 \sin(dx+c) / (\cos(dx+c)+1) + 1) + 837 \cdot \log(2 \sin(dx+c) / (\cos(dx+c)+1) - 1)) / d$

6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d
) - 8940*tan(c/2 + d*x/2)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/
 2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d), True))

Giac [A]

time = 0.45, size = 97, normalized size = 0.70

$$\frac{20 \left(2832 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1696 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 447 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1 \right)^3} - 837 \log\left(\left| 2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right|\right) + 837 \log\left(\left| 2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right|\right)$$

$$98304 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+5*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/98304*(20*(2832*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 4
 47*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 - 1)^3 - 837*log(abs(2*t
 an(1/2*d*x + 1/2*c) + 1)) + 837*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d

Mupad [B]

time = 0.00, size = 102, normalized size = 0.74

$$\frac{279 \operatorname{atanh}\left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{16384 d} - \frac{\frac{295 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{32768} - \frac{265 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{49152} + \frac{745 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{524288}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{4} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{16} - \frac{1}{64} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*cos(c + d*x) - 3)^4,x)

[Out] (279*atanh(2*tan(c/2 + (d*x)/2)))/(16384*d) - ((745*tan(c/2 + (d*x)/2))/524
 288 - (265*tan(c/2 + (d*x)/2)^3)/49152 + (295*tan(c/2 + (d*x)/2)^5)/32768)/
 (d*((3*tan(c/2 + (d*x)/2)^2)/16 - (3*tan(c/2 + (d*x)/2)^4)/4 + tan(c/2 + (d
 *x)/2)^6 - 1/64))

$$3.46 \quad \int \frac{1}{-3-5 \cos(c+dx)} dx$$

Optimal. Leaf size=65

$$\frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

[Out] 1/4*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-1/4*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 213}

$$\frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right) + 2 \cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(-3 - 5*Cos[c + d*x])^(-1), x]

[Out] Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(4*d) - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(4*d)

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{-3-5 \cos(c+dx)} dx &= \frac{2 \text{Subst}\left(\int \frac{1}{-8+2x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} \\ &= \frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 65, normalized size = 1.00

$$\frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

`[In] Integrate[(-3 - 5*Cos[c + d*x])^(-1), x]``[Out] Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(4*d) - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(4*d)`**Maple [A]**

time = 0.05, size = 34, normalized size = 0.52

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{4} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{4}}{d}$	34
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{4} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{4}}{d}$	34
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{4d} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{4d}$	36
risch	$\frac{\ln\left(e^{i(dx+c)} + \frac{3}{5} - \frac{4i}{5}\right)}{4d} - \frac{\ln\left(e^{i(dx+c)} + \frac{3}{5} + \frac{4i}{5}\right)}{4d}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-3-5*cos(d*x+c)), x, method=_RETURNVERBOSE)``[Out] 1/d*(-1/4*ln(tan(1/2*d*x+1/2*c)+2)+1/4*ln(tan(1/2*d*x+1/2*c)-2))`**Maxima [A]**

time = 0.30, size = 48, normalized size = 0.74

$$\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2\right) - \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(-3-5*cos(d*x+c)), x, algorithm="maxima")``[Out] -1/4*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d`**Fricas [A]**

time = 0.38, size = 46, normalized size = 0.71

$$\frac{\log\left(\frac{3}{2}\cos(dx+c) + 2\sin(dx+c) + \frac{5}{2}\right) - \log\left(\frac{3}{2}\cos(dx+c) - 2\sin(dx+c) + \frac{5}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c)),x, algorithm="fricas")

[Out] $-1/8*(\log(3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - \log(3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2))/d$

Sympy [A]

time = 0.33, size = 42, normalized size = 0.65

$$\begin{cases} \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 2\right)}{4d} - \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 2\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{-5\cos(c) - 3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c)),x)

[Out] Piecewise((log(tan(c/2 + d*x/2) - 2)/(4*d) - log(tan(c/2 + d*x/2) + 2)/(4*d)), Ne(d, 0)), (x/(-5*cos(c) - 3), True))

Giac [A]

time = 0.41, size = 34, normalized size = 0.52

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2\right|\right) - \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2\right|\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c)),x, algorithm="giac")

[Out] $-1/4*(\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 2)) - \log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 2)))/d$

Mupad [B]

time = 0.26, size = 17, normalized size = 0.26

$$-\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(5*cos(c + d*x) + 3),x)

[Out] $-\operatorname{atanh}(\tan(c/2 + (d*x)/2)/2)/(2*d)$

$$3.47 \quad \int \frac{1}{(-3-5 \cos(c+dx))^2} dx$$

Optimal. Leaf size=90

$$\frac{3 \log(2 \cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx)))}{64d} - \frac{3 \log(2 \cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx)))}{64d} + \frac{5 \sin(c+dx)}{16d(3+5 \cos(c+dx))}$$

[Out] 3/64*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/64*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/16*sin(d*x+c)/d/(3+5*cos(d*x+c))

Rubi [A]

time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {2743, 12, 2738, 213}

$$\frac{5 \sin(c+dx)}{16d(5 \cos(c+dx)+3)} + \frac{3 \log(2 \cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx)))}{64d} - \frac{3 \log(\sin(\frac{1}{2}(c+dx)) + 2 \cos(\frac{1}{2}(c+dx)))}{64d}$$

Antiderivative was successfully verified.

[In] Int[(-3 - 5*Cos[c + d*x])^(-2),x]

[Out] (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(64*d) - (3*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(64*d) + (5*Sin[c + d*x])/(16*d*(3 + 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-3 - 5 \cos(c + dx))^2} dx &= \frac{5 \sin(c + dx)}{16d(3 + 5 \cos(c + dx))} + \frac{1}{16} \int \frac{3}{-3 - 5 \cos(c + dx)} dx \\
&= \frac{5 \sin(c + dx)}{16d(3 + 5 \cos(c + dx))} + \frac{3}{16} \int \frac{1}{-3 - 5 \cos(c + dx)} dx \\
&= \frac{5 \sin(c + dx)}{16d(3 + 5 \cos(c + dx))} + \frac{3 \text{Subst}\left(\int \frac{1}{-8+2x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{8d} \\
&= \frac{3 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d} - \frac{3 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{64d}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 143, normalized size = 1.59

$$\frac{9 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 15 \cos(c + dx) \left(\log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) - 9 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right) + 20 \sin(c + dx)}{64d(3 + 5 \cos(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 5*Cos[c + d*x])^(-2), x]
```

```
[Out] (9*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 15*Cos[c + d*x]*(Log[2*Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) - 9*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d
*(3 + 5*Cos[c + d*x]))
```

Maple [A]

time = 0.06, size = 64, normalized size = 0.71

method	result	size
derivativedivides	$ \frac{-\frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{64} - \frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{64}}{d} $	64
default	$ \frac{-\frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{64} - \frac{5}{32 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{64}}{d} $	64
norman	$ -\frac{5 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16d \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 4\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{64d} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{64d} $	65

risch	$\frac{i(3e^{i(dx+c)}+5)}{8d(5e^{2i(dx+c)}+6e^{i(dx+c)}+5)} - \frac{3\ln(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5})}{64d} + \frac{3\ln(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5})}{64d}$	85
-------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3-5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out] $1/d*(-5/32/(\tan(1/2*d*x+1/2*c)+2)-3/64*\ln(\tan(1/2*d*x+1/2*c)+2)-5/32/(\tan(1/2*d*x+1/2*c)-2)+3/64*\ln(\tan(1/2*d*x+1/2*c)-2))$

Maxima [A]

time = 0.29, size = 91, normalized size = 1.01

$$\frac{\frac{20 \sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 4\right)(\cos(dx+c)+1)} + 3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2\right) - 3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3-5*cos(d*x+c))^2,x, algorithm="maxima")`

[Out] $-1/64*(20*\sin(d*x + c)/((\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 - 4)*(\cos(d*x + c) + 1)) + 3*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 2) - 3*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 2))/d$

Fricas [A]

time = 0.35, size = 88, normalized size = 0.98

$$\frac{3(5 \cos(dx+c) + 3) \log\left(\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 3(5 \cos(dx+c) + 3) \log\left(\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) - 40 \sin(dx+c)}{128(5d \cos(dx+c) + 3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3-5*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] $-1/128*(3*(5*\cos(d*x + c) + 3)*\log(3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - 3*(5*\cos(d*x + c) + 3)*\log(3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) - 40*\sin(d*x + c))/(5*d*\cos(d*x + c) + 3*d)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

time = 0.69, size = 231, normalized size = 2.57

$$\left\{ \begin{array}{ll} \frac{x}{(-3-5\cos(2\operatorname{atan}(2)))^2} & \text{for } c = -dx - 2\operatorname{atan}(2) \vee c = -dx + 2\operatorname{atan}(2) \\ \frac{x}{(-5\cos(c)-3)^2} & \text{for } d = 0 \\ \frac{3\log\left(\tan\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{12\log\left(\tan\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{3\log\left(\tan\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{12\log\left(\tan\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{64d\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{20\tan\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{\frac{c}{2}+\frac{dx}{2}\right)-256d}}{64d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3-5*cos(d*x+c))**2,x)`

[Out] Piecewise((x/(-3 - 5*cos(2*atan(2)))**2, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*x + 2*atan(2))), (x/(-5*cos(c) - 3)**2, Eq(d, 0)), (3*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 12*log(tan(c/2 + d*x/2) - 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 3*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) + 12*log(tan(c/2 + d*x/2) + 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 20*tan(c/2 + d*x/2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d), True))

Giac [A]

time = 0.46, size = 62, normalized size = 0.69

$$\frac{20 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 4} + 3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2\right|\right) - 3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2\right|\right)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 4) + 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) - 3*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d

Mupad [B]

time = 0.00, size = 47, normalized size = 0.52

$$-\frac{3 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{32 d} - \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*cos(c + d*x) + 3)^2,x)

[Out] - (3*atanh(tan(c/2 + (d*x)/2)/2))/(32*d) - (5*tan(c/2 + (d*x)/2))/(16*d*(tan(c/2 + (d*x)/2)^2 - 4))

$$3.48 \quad \int \frac{1}{(-3-5 \cos(c+dx))^3} dx$$

Optimal. Leaf size=115

$$\frac{43 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right)}{2048d} - \frac{43 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) + \sin \left(\frac{1}{2}(c+dx) \right) \right)}{2048d} - \frac{5 \sin(c+dx)}{32d(3+5 \cos(c+dx))}$$

[Out] 43/2048*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-43/2048*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-5/32*sin(d*x+c)/d/(3+5*cos(d*x+c))^2+45/512*sin(d*x+c)/d/(3+5*cos(d*x+c))

Rubi [A]

time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$,

Rules used = {2743, 2833, 12, 2738, 213}

$$\frac{45 \sin(c+dx)}{512d(5 \cos(c+dx)+3)} - \frac{5 \sin(c+dx)}{32d(5 \cos(c+dx)+3)^2} + \frac{43 \log \left(2 \cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right)}{2048d} - \frac{43 \log \left(\sin \left(\frac{1}{2}(c+dx) \right) + 2 \cos \left(\frac{1}{2}(c+dx) \right) \right)}{2048d}$$

Antiderivative was successfully verified.

[In] Int[(-3 - 5*Cos[c + d*x])^(-3), x]

[Out] (43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(2048*d) - (43*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(2048*d) - (5*Sin[c + d*x])/(32*d*(3 + 5*Cos[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-3 - 5 \cos(c + dx))^3} dx &= -\frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} + \frac{1}{32} \int \frac{6 - 5 \cos(c + dx)}{(-3 - 5 \cos(c + dx))^2} dx \\
&= -\frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))} + \frac{1}{512} \int \frac{43}{-3 - 5 \cos(c + dx)} dx \\
&= -\frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))} + \frac{43}{512} \int \frac{1}{-3 - 5 \cos(c + dx)} dx \\
&= -\frac{5 \sin(c + dx)}{32d(3 + 5 \cos(c + dx))^2} + \frac{45 \sin(c + dx)}{512d(3 + 5 \cos(c + dx))} + \frac{43 \text{Subst}\left(\int \frac{1}{-8 + 2x^2} dx, \frac{1}{-8 + 2x^2}\right)}{256} \\
&= \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} - \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 217, normalized size = 1.89

$$\frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} - \frac{43 \log\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2048d} - \frac{5}{512d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{2048d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} + \frac{5}{512d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{45 \sin\left(\frac{1}{2}(c + dx)\right)}{2048d\left(2 \cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 5*Cos[c + d*x])^(-3), x]
```

```
[Out] (43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(2048*d) - (43*Log[2*Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]/(2048*d) - 5/(512*d*(2*Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^2) + (45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] - S
in[(c + d*x)/2])) + 5/(512*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (
45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]

time = 0.07, size = 94, normalized size = 0.82

method	result
norman	$\frac{35 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 85 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{128d} - \frac{512d}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 4\right)^2} + \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{2048d} - \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{2048d}$
derivativedivides	$\frac{25}{512\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} - \frac{85}{1024\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{2048} - \frac{25}{512\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^2} - \frac{85}{1024\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} + \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{2048}$
default	$\frac{25}{512\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} - \frac{85}{1024\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{2048} - \frac{25}{512\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^2} - \frac{85}{1024\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)} + \frac{43 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{2048}$
risch	$\frac{i(215 e^{3i(dx+c)} + 387 e^{2i(dx+c)} + 325 e^{i(dx+c)} + 225)}{256d(5 e^{2i(dx+c)} + 6 e^{i(dx+c)} + 5)^2} + \frac{43 \ln\left(e^{i(dx+c)} + \frac{3}{5} - \frac{4i}{5}\right)}{2048d} - \frac{43 \ln\left(e^{i(dx+c)} + \frac{3}{5} + \frac{4i}{5}\right)}{2048d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3-5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] $1/d*(25/512/(\tan(1/2*d*x+1/2*c)+2)^2-85/1024/(\tan(1/2*d*x+1/2*c)+2)-43/2048*\ln(\tan(1/2*d*x+1/2*c)+2)-25/512/(\tan(1/2*d*x+1/2*c)-2)^2-85/1024/(\tan(1/2*d*x+1/2*c)-2)+43/2048*\ln(\tan(1/2*d*x+1/2*c)-2))$

Maxima [A]

time = 0.27, size = 135, normalized size = 1.17

$$\frac{20 \left(\frac{28 \sin(dx+c)}{\cos(dx+c)+1} - \frac{17 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{\frac{8 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 16} + 43 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2 \right) - 43 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2 \right)$$

$$2048 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3-5*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $-1/2048*(20*(28*\sin(d*x + c)/(\cos(d*x + c) + 1) - 17*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(8*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 - \sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 - 16) + 43*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 2) - 43*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 2))/d$

Fricas [A]

time = 0.37, size = 129, normalized size = 1.12

$$\frac{43(25 \cos(dx+c)^2 + 30 \cos(dx+c) + 9) \log\left(\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 43(25 \cos(dx+c)^2 + 30 \cos(dx+c) + 9) \log\left(\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) - 40(45 \cos(dx+c) + 11) \sin(dx+c)}{4096(25d \cos(dx+c)^2 + 30d \cos(dx+c) + 9d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3-5*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $-1/4096*(43*(25*\cos(d*x + c)^2 + 30*\cos(d*x + c) + 9)*\log(3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - 43*(25*\cos(d*x + c)^2 + 30*\cos(d*x + c) + 9)*\log($

$3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) - 40*(45*\cos(d*x + c) + 11)*\sin(d*x + c))/(25*d*\cos(d*x + c)^2 + 30*d*\cos(d*x + c) + 9*d)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(102) = 204$.

time = 1.27, size = 478, normalized size = 4.16

$$\frac{\int \frac{3 \cos(dx + c) - 2 \sin(dx + c) + 5/2 - 40(45 \cos(dx + c) + 11) \sin(dx + c)}{25d \cos^2(dx + c) + 30d \cos(dx + c) + 9d} dx}{\int \frac{3 \cos(dx + c) - 2 \sin(dx + c) + 5/2 - 40(45 \cos(dx + c) + 11) \sin(dx + c)}{25d \cos^2(dx + c) + 30d \cos(dx + c) + 9d} dx} \quad \text{for } c = -dx - 2\operatorname{atan}(2) \vee c = -dx + 2\operatorname{atan}(2) \text{ for } d = 0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c))**3,x)

[Out] Piecewise((x/(-3 - 5*cos(2*atan(2)))**3, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*x + 2*atan(2))), (x/(-5*cos(c) - 3)**3, Eq(d, 0)), (43*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**4/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 344*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 688*log(tan(c/2 + d*x/2) - 2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 43*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 344*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 688*log(tan(c/2 + d*x/2) + 2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 340*tan(c/2 + d*x/2)**3/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 560*tan(c/2 + d*x/2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d), True))

Giac [A]

time = 0.45, size = 78, normalized size = 0.68

$$\frac{20 \left(17 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 28 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 4 \right)^2} + 43 \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2 \right|\right) - 43 \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2 \right|\right)}{2048 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c))^3,x, algorithm="giac")

[Out] $-1/2048*(20*(17*\tan(1/2*d*x + 1/2*c)^3 - 28*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 4)^2 + 43*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) + 2)) - 43*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) - 2)))/d$

Mupad [B]

time = 0.67, size = 75, normalized size = 0.65

$$\frac{\frac{35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{128} - \frac{85 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{512}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 8 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 16 \right)} - \frac{43 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{1024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(5*cos(c + d*x) + 3)^3,x)`

[Out]
$$\frac{((35*\tan(c/2 + (d*x)/2))/128 - (85*\tan(c/2 + (d*x)/2)^3)/512)/(d*(\tan(c/2 + (d*x)/2)^4 - 8*\tan(c/2 + (d*x)/2)^2 + 16)) - (43*atanh(\tan(c/2 + (d*x)/2))/2))/(1024*d)}$$

3.49 $\int \frac{1}{(-3-5\cos(c+dx))^4} dx$

Optimal. Leaf size=140

$$\frac{279 \log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279 \log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5 \sin(c+dx)}{48d(3+5\cos(c+dx))}$$

[Out] 279/32768*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-279/32768*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/48*sin(d*x+c)/d/(3+5*cos(d*x+c))^3-25/512*sin(d*x+c)/d/(3+5*cos(d*x+c))^2+995/24576*sin(d*x+c)/d/(3+5*cos(d*x+c))

Rubi [A]

time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 213}

$$\frac{995 \sin(c+dx)}{24576d(5 \cos(c+dx)+3)} - \frac{25 \sin(c+dx)}{512d(5 \cos(c+dx)+3)^2} + \frac{5 \sin(c+dx)}{48d(5 \cos(c+dx)+3)^3} + \frac{279 \log\left(2 \cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279 \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + 2 \cos\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

Antiderivative was successfully verified.

[In] Int[(-3 - 5*Cos[c + d*x])^(-4), x]

[Out] (279*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(32768*d) - (279*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(32768*d) + (5*Sin[c + d*x])/(48*d*(3 + 5*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x])^2) + (995*Sin[c + d*x])/(24576*d*(3 + 5*Cos[c + d*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 376650*Cos[2*(c + d*x)]*(Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 467046*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 226140*Sin[c + d*x] + 190800*Sin[2*(c + d*x)] + 99500*Sin[3*(c + d*x)])/(393216*d*(3 + 5*Cos[c + d*x])^3)

Maple [A]

time = 0.09, size = 124, normalized size = 0.89

method	result
norman	$\frac{-\frac{295 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{265 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{768d} - \frac{745 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8192d}}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 4\right)^3} + \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)}{32768d} - \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{32768d}$
derivativedivides	$\frac{\frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^3} + \frac{175}{4096 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} - \frac{745}{16384 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{32768} - \frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^3}}{d}$
default	$\frac{\frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^3} + \frac{175}{4096 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)^2} - \frac{745}{16384 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)} - \frac{279 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)}{32768} - \frac{125}{6144 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\right)^3}}{d}$
risch	$\frac{i(20925 e^{5i(dx+c)} + 62775 e^{4i(dx+c)} + 111042 e^{3i(dx+c)} + 119310 e^{2i(dx+c)} + 68625 e^{i(dx+c)} + 24875)}{12288d(5 e^{2i(dx+c)} + 6 e^{i(dx+c)} + 5)^3} + \frac{279 \ln(e^{i(dx+c)} + \frac{3}{5})}{32768d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3-5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-125/6144/(tan(1/2*d*x+1/2*c)+2)^3+175/4096/(tan(1/2*d*x+1/2*c)+2)^2-745/16384/(tan(1/2*d*x+1/2*c)+2)-279/32768*ln(tan(1/2*d*x+1/2*c)+2)-125/6144/(tan(1/2*d*x+1/2*c)-2)^3-175/4096/(tan(1/2*d*x+1/2*c)-2)^2-745/16384/(tan(1/2*d*x+1/2*c)-2)+279/32768*ln(tan(1/2*d*x+1/2*c)-2))

Maxima [A]

time = 0.27, size = 174, normalized size = 1.24

$$\frac{20 \left(\frac{2832 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1696 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{447 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) + 837 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2 \right) - 837 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2 \right)}{\frac{48 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{12 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - 64} 98304 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/98304*(20*(2832*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 447*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) + 837*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 837*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d

Fricas [A]

time = 0.37, size = 170, normalized size = 1.21

$$\frac{837(125 \cos(dx+c)^3 + 225 \cos(dx+c)^2 + 135 \cos(dx+c) + 27) \log\left(\frac{3}{2} \cos(dx+c) + 2 \sin(dx+c) + \frac{5}{2}\right) - 837(125 \cos(dx+c)^3 + 225 \cos(dx+c)^2 + 135 \cos(dx+c) + 27) \log\left(\frac{3}{2} \cos(dx+c) - 2 \sin(dx+c) + \frac{5}{2}\right) - 40(4975 \cos(dx+c)^2 + 4770 \cos(dx+c) + 1583) \sin(dx+c)}{196608(125d \cos(dx+c)^3 + 225d \cos(dx+c)^2 + 135d \cos(dx+c) + 27d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\frac{-1/196608*(837*(125*\cos(d*x + c)^3 + 225*\cos(d*x + c)^2 + 135*\cos(d*x + c) + 27)*\log(3/2*\cos(d*x + c) + 2*\sin(d*x + c) + 5/2) - 837*(125*\cos(d*x + c)^3 + 225*\cos(d*x + c)^2 + 135*\cos(d*x + c) + 27)*\log(3/2*\cos(d*x + c) - 2*\sin(d*x + c) + 5/2) - 40*(4975*\cos(d*x + c)^2 + 4770*\cos(d*x + c) + 1583)*\sin(d*x + c)}{(125*d*\cos(d*x + c)^3 + 225*d*\cos(d*x + c)^2 + 135*d*\cos(d*x + c) + 27*d)}$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(126) = 252.

time = 2.69, size = 816, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-5*cos(d*x+c))**4,x)

[Out] Piecewise((x/(-3 - 5*cos(2*atan(2)))**4, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*x + 2*atan(2))), (x/(-5*cos(c) - 3)**4, Eq(d, 0)), (837*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 10044*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 40176*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 53568*log(tan(c/2 + d*x/2) - 2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 837*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 10044*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 40176*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 53568*log(tan(c/2 + d*x/2) + 2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 8940*tan(c/2 + d*x/2)**5/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 33920*tan(c/2 + d*x/2)**3/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d

```
*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 56640*tan(c/2 + d*x/2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d), True))
```

Giac [A]

time = 0.46, size = 91, normalized size = 0.65

$$\frac{20 \left(447 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1696 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 2832 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 4 \right)^3} + 837 \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2 \right|\right) - 837 \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2 \right|\right)$$

98304 d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-5*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(20*(447*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 2832*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^3 + 837*log(abs(tan(1/2*d*x + 1/2*c) + 2)) - 837*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B]

time = 0.00, size = 102, normalized size = 0.73

$$\frac{279 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}\right)}{16384 d} - \frac{\frac{745 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8192} - \frac{265 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{768} + \frac{295 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{512}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 12 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 48 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 64 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cos(c + d*x) + 3)^4,x)
```

```
[Out] - (279*atanh(tan(c/2 + (d*x)/2)/2))/(16384*d) - ((295*tan(c/2 + (d*x)/2))/512 - (265*tan(c/2 + (d*x)/2)^3)/768 + (745*tan(c/2 + (d*x)/2)^5)/8192)/(d*(48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 64))
```

3.50 $\int (a + b \cos(c + dx))^{5/2} dx$

Optimal. Leaf size=197

$$\frac{2(23a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{16a(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d \sqrt{a + b \cos(c + dx)}}$$

[Out] $2/5*b*(a+b*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+16/15*a*b*\sin(d*x+c)*(a+b*\cos(d*x+c))^(1/2)/d+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*\cos(d*x+c))^(1/2)/d/((a+b*\cos(d*x+c))/(a+b))^(1/2)-16/15*a*(a^2-b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*\cos(d*x+c))/(a+b))^(1/2)/d/(a+b*\cos(d*x+c))^(1/2)$

Rubi [A]

time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{16a(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d \sqrt{a + b \cos(c + dx)}} + \frac{2(23a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} + \frac{2b \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{5d} + \frac{16ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{15d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(5/2), x]

[Out] $(2*(23*a^2 + 9*b^2)*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*d*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]) + (16*a*b*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*d) + (2*b*(a + b*\text{Cos}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(5*d)$

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2735

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int (a + b \cos(c + dx))^{5/2} dx &= \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2}{5} \int \sqrt{a + b \cos(c + dx)} \left(\frac{1}{2}(5a^2 + 3b^2) \right. \\
&= \frac{16ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&= \frac{16ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&= \frac{16ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&= \frac{2(23a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 16a(a^2 - b^2) \sqrt{\frac{a+b}{a+b}}}{15d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{16a(a^2 - b^2) \sqrt{\frac{a+b}{a+b}}}{15d \sqrt{a+b}}
\end{aligned}$$

Mathematica [A]

time = 0.55, size = 177, normalized size = 0.90

$$\frac{2(23a^3 + 23a^2b + 9ab^2 + 9b^3) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 16a(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) + b(22a^2 + 3b^2 + 28ab \cos(c + dx) + 3b^2 \cos(2(c + dx))) \sin(c + dx)}{15d \sqrt{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*Cos[c + d*x])^(5/2), x]`

```
[Out] (2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])]/(a + b)]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c
+ d*x])]/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2
+ 28*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x]/(15*d*Sqrt[a
+ b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(235) = 470.

time = 0.17, size = 662, normalized size = 3.36

method	result
--------	--------


```
[Out] 1/45*(sqrt(2)*(I*a^3 - 33*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-I*a^3 + 33*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-23*I*a^2*b - 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(23*I*a^2*b + 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(3*b^3*cos(d*x + c) + 11*a*b^2)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c)/(b*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cos(c + dx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(5/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2), x)
```

3.51 $\int (a + b \cos(c + dx))^{3/2} dx$

Optimal. Leaf size=157

$$\frac{8a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 2(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) + 2b \sqrt{a + b \cos(c + dx)}}{3d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] $2/3*b*\sin(d*x+c)*(a+b*\cos(d*x+c))^(1/2)/d+8/3*a*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*\cos(d*x+c))^(1/2)/d/((a+b*\cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*\cos(d*x+c))/(a+b))^(1/2)/d/(a+b*\cos(d*x+c))^(1/2)$

Rubi [A]

time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$,

Rules used = {2735, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) + 2b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d \sqrt{a + b \cos(c + dx)}} + \frac{8a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Cos}[c + d*x])^(3/2), x]$

[Out] $(8*a*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*b)/(a + b)])/(3*d*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*b)/(a + b)])/(3*d*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]) + (2*b*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2732

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[2*(\text{Sqrt}[a + b]/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2*(b/(a + b))], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2734

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b/(a + b))*\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ !\text{GtQ}[a + b, 0]$

Rule 2735

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \cos(c + dx))^{3/2} dx &= \frac{2b \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}(3a^2 + b^2) + 2ab \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2b \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}(4a) \int \sqrt{a + b \cos(c + dx)} dx + \frac{1}{3}(-a) \\
&= \frac{2b \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(4a \sqrt{a + b \cos(c + dx)}) \int \sqrt{\frac{a}{a+b} + \frac{b}{a+b} \cos(c + dx)}}{3 \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
&= \frac{8a \sqrt{a + b \cos(c + dx)} E(\frac{1}{2}(c + dx) | \frac{2b}{a+b})}{3d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{2(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}}}{3d \sqrt{a + b \cos(c + dx)}} F(\frac{1}{2}(c + dx) | \frac{2b}{a+b}) + 2b(a + b \cos(c + dx)) \sin(c + dx)
\end{aligned}$$

Mathematica [A]

time = 0.37, size = 134, normalized size = 0.85

$$\frac{8a(a+b) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E(\frac{1}{2}(c+dx) | \frac{2b}{a+b}) - 2(a^2-b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} F(\frac{1}{2}(c+dx) | \frac{2b}{a+b}) + 2b(a+b \cos(c+dx)) \sin(c+dx)}{3d \sqrt{a+b \cos(c+dx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*Cos[c + d*x])^(3/2), x]`

```
[Out] (8*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x]/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(199) = 398.

time = 0.12, size = 450, normalized size = 2.87

method	result
default	$ \frac{2 \sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b + a - b\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(4 \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^2 + 2 \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) ab - 6 \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b\right)} $

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^{2*b+a-b}*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(4*\cos(1/2*d*x+1/2*c)^5*b^2+2*\cos(1/2*d*x+1/2*c)^3*a*b-6*\cos(1/2*d*x+1/2*c)^3*b^2-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*\cos(1/2*d*x+1/2*c)^{2*b+a-b}/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2+(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*\cos(1/2*d*x+1/2*c)^{2*b+a-b}/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*b^2+4*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*\cos(1/2*d*x+1/2*c)^{2*b+a-b}/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)}))*a^2-4*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*\cos(1/2*d*x+1/2*c)^{2*b+a-b}/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a*b-2*\cos(1/2*d*x+1/2*c)*a*b+2*\cos(1/2*d*x+1/2*c)*b^2)/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^{2*b+a+b})^{(1/2)}/d$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.13, size = 398, normalized size = 2.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out]
$$1/9*(12*I*\sqrt{2}*a*b^{(3/2)}*\text{weierstrassZeta}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, \text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b)) - 12*I*\sqrt{2}*a*b^{(3/2)}*\text{weierstrassZeta}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, \text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b)) + 6*\sqrt{b*\cos(d*x + c) + a}*b^2*\sin(d*x + c) + \sqrt{2}*(-I*a^2 - 3*I*b^2)*\sqrt{b}*\text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b) + \sqrt{2}*(I*a^2 + 3*I*b^2)*\sqrt{b}*\text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b))/b*d$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cos(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(3/2),x)

[Out] Integral((a + b*cos(c + d*x))**(3/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x))^(3/2),x)

[Out] int((a + b*cos(c + d*x))^(3/2), x)

3.52 $\int \sqrt{a + b \cos(c + dx)} dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)}*(a+b*\cos(d*x+c))^{(1/2)}/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)})$

Rubi [A]

time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2734, 2732}

$$\frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Cos[c + d*x]],x]

[Out] $(2*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*b)/(a + b)])/(d*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)])$

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rubi steps

$$\int \sqrt{a + b \cos(c + dx)} dx = \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{\sqrt{\frac{a + b \cos(c + dx)}{a+b}}}$$

$$= \frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a + b \cos(c + dx)}{a+b}}}$$

Mathematica [A]

time = 0.05, size = 57, normalized size = 1.00

$$\frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a + b \cos(c + dx)}{a+b}}}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[a + b*Cos[c + d*x]],x]``[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(82) = 164.

time = 0.26, size = 170, normalized size = 2.98

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a - b\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a - b}{a - b}}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + (a + b)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b}}$

risch	$-\frac{i\sqrt{2} \sqrt{(b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b) e^{-i(dx+c)}}}{d}$	$i \frac{2a \left(a + \sqrt{a^2 - b^2} \right) \sqrt{\frac{\left(e^{i(dx+c)} + \frac{a + \sqrt{a^2 - b^2}}{b} \right) b}{a + \sqrt{a^2 - b^2}}}}{a + \sqrt{a^2 - b^2}}$
-------	--	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-2 * ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 * b + a - b) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 * b + a - b) / (a - b)) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b)) ^ (1/2)) * (a - b) / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / \sin(1/2 * d * x + 1/2 * c) / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 * b + a + b) ^ (1/2) / d$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*cos(d*x + c) + a), x)`

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.13, size = 355, normalized size = 6.23

$-\frac{1}{312} \sqrt{2} \sqrt{\text{atan2}\left(\frac{11d^2-35d}{11d^2-35d}, \frac{11d^2-35d}{11d^2-35d}\right)} + \frac{1}{312} \sqrt{2} \sqrt{\text{atan2}\left(\frac{11d^2-35d}{11d^2-35d}, \frac{11d^2-35d}{11d^2-35d}\right)} + \frac{1}{312} \sqrt{2} \sqrt{\text{atan2}\left(\frac{11d^2-35d}{11d^2-35d}, \frac{11d^2-35d}{11d^2-35d}\right)} - \frac{1}{312} \sqrt{2} \sqrt{\text{atan2}\left(\frac{11d^2-35d}{11d^2-35d}, \frac{11d^2-35d}{11d^2-35d}\right)} - \frac{1}{312} \sqrt{2} \sqrt{\text{atan2}\left(\frac{11d^2-35d}{11d^2-35d}, \frac{11d^2-35d}{11d^2-35d}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

```
[Out] 1/3*(-I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b) + I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
) + 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3
*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/(b*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + b \cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2), x)
```

$$3.53 \quad \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal. Leaf size=57

$$\frac{2\sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}}$$

[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2742, 2740}

$$\frac{2\sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx = \frac{\int \frac{1}{\sqrt{\frac{a + b \cos(c + dx)}{a + b}}} dx}{\sqrt{a + b \cos(c + dx)}}$$

$$= \frac{2 \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

Mathematica [A]

time = 0.04, size = 57, normalized size = 1.00

$$\frac{2 \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]],x]``[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.14, size = 75, normalized size = 1.32

method	result	size
default	$\frac{2 \sqrt{\frac{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a - b}{a + b}} \operatorname{am}^{-1} \left(\frac{dx}{2} + \frac{c}{2} \middle \frac{\sqrt{2} \sqrt{b}}{\sqrt{a + b}} \right)}{d \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a - b}}$	75

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)``[Out] 2/d/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.10, size = 146, normalized size = 2.56

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)+2a}{3b}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $(-I*\sqrt{2}*\sqrt{b}*\operatorname{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b) + I*\sqrt{2}*\sqrt{b}*\operatorname{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b))/(b*d)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x)

[Out] Integral(1/sqrt(a + b*cos(c + d*x)), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)

Mupad [B]

time = 0.39, size = 52, normalized size = 0.91

$$\frac{2F\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) \sqrt{\frac{a + b \cos(c + dx)}{a + b}}}{d \sqrt{a + b \cos(c + dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x))^(1/2),x)

[Out] $(2*\operatorname{ellipticF}(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*\cos(c + d*x))/(a + b))^(1/2))/(d*(a + b*\cos(c + d*x))^(1/2))$

3.54 $\int \frac{1}{(a+b \cos(c+dx))^{3/2}} dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2b \sin(c+dx)}{(a^2-b^2)d\sqrt{a+b \cos(c+dx)}}$$

[Out] $-2*b*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{(1/2)+2*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)*(b/(a+b))^{(1/2)})*(a+b*\cos(d*x+c))^{(1/2)/(a^2-b^2)/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2743, 21, 2734, 2732}

$$\frac{2\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d(a^2-b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2b \sin(c+dx)}{d(a^2-b^2)\sqrt{a+b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Cos}[c + d*x])^{(-3/2)}, x]$

[Out] $(2*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*b)/(a + b)])/((a^2 - b^2)*d*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]) - (2*b*\text{Sin}[c + d*x])/((a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Cos}[c + d*x]])$

Rule 21

$\text{Int}[(u_*)*((a_*) + (b_*)*(v_))^{(m_*)}*((c_*) + (d_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

Rule 2732

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Simp}[2*(\text{Sqrt}[a + b]/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2*(b/(a + b))], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2734

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b$

`/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]`

Rule 2743

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*SIN[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*SIN[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]`

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx &= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2 \int \frac{-\frac{a}{2} - \frac{1}{2} b \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a^2 - b^2} \\ &= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\int \sqrt{a + b \cos(c + dx)} dx}{a^2 - b^2} \\ &= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a}}}{(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\ &= \frac{2 \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right)}{(a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 83, normalized size = 0.78

$$\frac{2(a + b) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right) - 2b \sin(c + dx)}{(a - b)(a + b) d \sqrt{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*Cos[c + d*x])^(-3/2), x]`

`[Out] (2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*b*SIN[c + d*x])/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])`

Maple [A]

time = 0.12, size = 217, normalized size = 2.05

method	result
default	$\frac{2 \left(2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^b + \text{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a-b} + \frac{a+b}{a-b}} \right)}{(a-b)(a+b) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.14, size = 482, normalized size = 4.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + (I*sqrt(2)*a*b*cos(d*x + c) + I*sqrt(2)*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (-I*sqrt(2)*a*b*cos(d*x + c) - I*sqrt(2)*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(I*sqrt(2)*b^2*cos(d*x + c) + I*sqrt(2)*a*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(
```


$-I\sqrt{2}*b^2*\cos(d*x + c) - I\sqrt{2}*a*b*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b)))/((a^2*b^2 - b^4)*d*\cos(d*x + c) + (a^3*b - a*b^3)*d)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral((a + b*cos(c + d*x))**(-3/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x))^(3/2),x)

[Out] int(1/(a + b*cos(c + d*x))^(3/2), x)

3.55 $\int \frac{1}{(a+b \cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=221

$$\frac{8a \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} - \frac{2b \sin(c+dx)}{3(a^2-b^2) d(a+b \cos(c+dx))}$$

[Out] $-2/3*b*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{3/2}-8/3*a*b*\sin(d*x+c)/(a^2-b^2)^2/d/(a+b*\cos(d*x+c))^{1/2}+8/3*a*(\cos(1/2*d*x+1/2*c))^2^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(b/(a+b))^{1/2})*(a+b*\cos(d*x+c))^{1/2}/(a^2-b^2)^2/d/((a+b*\cos(d*x+c))/(a+b))^{1/2}-2/3*(\cos(1/2*d*x+1/2*c))^2^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(b/(a+b))^{1/2})*((a+b*\cos(d*x+c))/(a+b))^{1/2}/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{1/2}$

Rubi [A]

time = 0.16, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{8ab \sin(c+dx)}{3d(a^2-b^2)^2 \sqrt{a+b \cos(c+dx)}} - \frac{2b \sin(c+dx)}{3d(a^2-b^2)(a+b \cos(c+dx))^{3/2}} - \frac{2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3d(a^2-b^2) \sqrt{a+b \cos(c+dx)}} + \frac{8a \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3d(a^2-b^2)^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Cos}[c + d*x])^{-5/2}, x]$

[Out] $(8*a*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)^2*d*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]) - (2*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]) - (2*b*\text{Sin}[c + d*x])/(3*(a^2 - b^2)*d*(a + b*\text{Cos}[c + d*x])^{3/2}) - (8*a*b*\text{Sin}[c + d*x])/(3*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Cos}[c + d*x]])$

Rule 2732

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[2*(\text{Sqrt}[a + b]/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2*(b/(a + b))], x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2734

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b$

$$\frac{1}{(a+b)\sin[c+dx]}, x, x \text{ /; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{!GtQ}[a + b, 0]$$

Rule 2740

$$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \text{ :> } \text{Simp}[(2/(d*\text{Sqrt}[a + b]))*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + dx), 2*(b/(a + b))], x] \text{ /; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$$

Rule 2742

$$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \text{ :> } \text{Dist}[\text{Sqrt}[(a + b*\sin[c + dx])/(a + b)]/\text{Sqrt}[a + b*\sin[c + dx]], \text{Int}[1/\text{Sqrt}[a/(a + b) + (b/(a + b))*\sin[c + dx]], x], x] \text{ /; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{!GtQ}[a + b, 0]$$

Rule 2743

$$\text{Int}[(a_.) + (b_.)\sin[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[(-b)*\text{Cos}[c + dx]*((a + b*\sin[c + dx])^{(n + 1)}/(d*(n + 1)*(a^2 - b^2))), x] + \text{Dist}[1/((n + 1)*(a^2 - b^2)), \text{Int}[(a + b*\sin[c + dx])^{(n + 1)}*\text{Simp}[a*(n + 1) - b*(n + 2)*\sin[c + dx], x], x], x] \text{ /; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$$

Rule 2831

$$\text{Int}[(c_.) + (d_.)\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(a_.) + (b_.)\sin[(e_.) + (f_.)*(x_)]], x_Symbol] \text{ :> } \text{Dist}[(b*c - a*d)/b, \text{Int}[1/\text{Sqrt}[a + b*\sin[e + f*x]], x], x] + \text{Dist}[d/b, \text{Int}[\text{Sqrt}[a + b*\sin[e + f*x]], x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$$

Rule 2833

$$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)*(x_)]^{(m_.)}*((c_.) + (d_.)\sin[(e_.) + (f_.)*(x_)]), x_Symbol] \text{ :> } \text{Simp}[(-b*c - a*d)*\text{Cos}[e + f*x]*((a + b*\sin[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 - b^2))), x] + \text{Dist}[1/((m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*\text{Simp}[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*\sin[e + f*x], x], x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*m]$$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx &= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{2 \int \frac{-\frac{3a}{2} + \frac{1}{2}b \cos(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx}{3(a^2 - b^2)} \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \dots \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \dots \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \dots \\
&= \frac{8a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3(a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{2 \sqrt{\frac{a + b \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A]

time = 0.62, size = 158, normalized size = 0.71

$$\frac{8a(a+b)^2 \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{3/2} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) - 2(a-b)(a+b)^2 \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{3/2} F\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) + 2b(-5a^2 + b^2 - 4ab \cos(c+dx)) \sin(c+dx)}{3(a-b)^2(a+b)^2 d(a+b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*Cos[c + d*x])^(-5/2), x]`

```
[Out] (8*a*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticE[(c + d*x)/2,
(2*b)/(a + b)] - 2*(a - b)*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*E
llipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cos[c + d*
x])*Sin[c + d*x])/(3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [A]

time = 0.20, size = 489, normalized size = 2.21

method	result
--------	--------

default	$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b - a + b\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + (a + b)}} \frac{3b(a-b)(a+b)\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{a-b}{2b}\right)}{}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -\left(-2\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2b - a + b\right)\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\left(\frac{1}{3}b/(a-b)\right) \\ & / (a+b)\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)\left(-2\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^4b + (a+b)\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right)^{1/2} \\ & / \left(\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 + \frac{1}{2}b/(a-b)\right)^2 + \frac{16}{3}\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 \\ & b/(a-b)^2/(a+b)^2\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)a / \left(-2\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2b - a + b\right)\sin \\ & \left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right)^{1/2} + 2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*\left(\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right)^{1/2} \\ & * \left(\left(2\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2b + a - b\right)/(a-b)\right)^{1/2} / \left(-2\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^4b + (a+b)\right) \\ & * \sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right)^{1/2} * \text{EllipticF}\left(\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right), \left(-2*b/(a-b)\right)^{1/2}\right) - 8/3*a/(a-b)/(a+b)^2 \\ & * \left(\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right)^{1/2} * \left(\left(2\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2b + a - b\right)/(a-b)\right)^{1/2} / \left(-2\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^4b + (a+b)\right) \\ & * \sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right)^{1/2} * \left(\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right), \left(-2*b/(a-b)\right)^{1/2}\right) - \text{EllipticE}\left(\cos\left(\frac{1}{2}d*x + \frac{1}{2}c\right), \left(-2*b/(a-b)\right)^{1/2}\right)\right) \\ & / \sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right) / \left(-2\sin\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2b + a + b\right)^{1/2} / d \end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(-5/2), x)`

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.15, size = 692, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & -1/9*(6*(4*a*b^3*\cos(d*x + c) + 5*a^2*b^2 - b^4)*\text{sqrt}(b*\cos(d*x + c) + a)*\sin(d*x + c) \\ & - (\text{sqrt}(2)*(-I*a^2*b^2 - 3*I*b^4)*\cos(d*x + c)^2 - 2*\text{sqrt}(2)*(I*a^3*b + 3*I*a*b^3)*\cos(d*x + c) + \text{sqrt}(2)*(-I*a^4 - 3*I*a^2*b^2))*\text{sqrt}(b)* \end{aligned}$$

```

weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(I*a^2*b^2 +
3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*cos(d*x + c) +
sqrt(2)*(I*a^4 + 3*I*a^2*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x
+ c) + 2*a)/b) + 12*(-I*sqrt(2)*a*b^3*cos(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2
*cos(d*x + c) - I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b)) + 12*(I*sqrt(2)*a*b^3*cos(d*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*co
s(d*x + c) + I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b
^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
+ 2*a)/b)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c)^2 + 2*(a^5*b^2 - 2*
a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)

```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral((a + b*cos(c + d*x))**(-5/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x))^(5/2),x)

[Out] int(1/(a + b*cos(c + d*x))^(5/2), x)

3.56 $\int (a + b \cos(c + dx))^{4/3} dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2} (a + b) F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{4}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] (a+b)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*
(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/
(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} (a + b) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{4}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(4/3),x]

[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x]))/(a + b))^(1/3)

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,

m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b *c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x) ^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx))^{4/3} dx &= -\frac{\sin(c + dx) \operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\ &= \frac{\left((-a - b)\sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)\right) \operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\sqrt[3]{-\frac{a + b \cos(c + dx)}{-a - b}}} \\ &= \frac{\sqrt{2}(a + b)F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{4}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)\sqrt[3]{a + b \cos(c + dx)}}{d\sqrt{1 + \cos(c + dx)}\sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 246 vs. 2(108) = 216.

time = 1.27, size = 246, normalized size = 2.28

$$\frac{3\sqrt{a + b \cos(c + dx)} \operatorname{csc}(c + dx) \left(4(-a^2 + b^2)F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{4}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{-\frac{b(1 + \cos(c + dx))}{a - b}} + 5aF_1\left(\frac{3}{2}; \frac{1}{2}, \frac{1}{2}; \frac{4}{3}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{-\frac{b(1 + \cos(c + dx))}{a - b}}\right) - 4b^2 \sin^2(c + dx)}{16bd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x])^(4/3), x]

[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + 5*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*(a + b*Cos[c + d*x]) - 4*b^2*Sin[c + d*x]^2)/(16*b*d)

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int (a + b \cos(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(4/3),x)

[Out] int((a+b*cos(d*x+c))^(4/3),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(4/3), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(4/3), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cos(c + dx))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(4/3),x)

[Out] Integral((a + b*cos(c + d*x))**(4/3), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(4/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((a + b*cos(c + d*x))^(4/3), x)
```

3.57 $\int (a + b \cos(c + dx))^{2/3} dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right) (a + b \cos(c + dx))^{2/3} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)} \left(\frac{a + b \cos(c + dx)}{a+b}\right)^{2/3}}$$

[Out] AppellF1(1/2, -2/3, 1/2, 3/2, b*(1-cos(d*x+c))/(a+b), 1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} \sin(c + dx)(a + b \cos(c + dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right)}{d \sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(2/3), x]

[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x]/(d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x))/(a + b))^(2/3))

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b

*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx))^{2/3} dx &= -\frac{\sin(c + dx) \operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\ &= -\frac{((a + b \cos(c + dx))^{2/3} \sin(c + dx)) \operatorname{Subst}\left(\int \frac{\left(\frac{-a}{-a-b} - \frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} \\ &= \frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right) (a + b \cos(c + dx))^{2/3} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 118, normalized size = 1.12

$$\frac{3F_1\left(\frac{5}{3}; \frac{1}{2}, \frac{1}{2}, \frac{8}{3}; \frac{a+b\cos(c+dx)}{a-b}, \frac{a+b\cos(c+dx)}{a+b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx))}{a+b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a+b}} (a + b \cos(c + dx))^{5/3} \csc(c + dx)}{5bd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x])^(2/3), x]

[Out] (-3*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(5/3)*Csc[c + d*x])/(5*b*d)

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int (a + b \cos(dx + c))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cos(d*x+c))^(2/3),x)`

[Out] `int((a+b*cos(d*x+c))^(2/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(2/3), x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((b*cos(d*x + c) + a)^(2/3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cos(c + dx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))**(2/3),x)`

[Out] `Integral((a + b*cos(c + d*x))**(2/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(2/3),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(2/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((a + b*cos(c + d*x))^(2/3), x)
```

3.58 $\int \sqrt[3]{a + b \cos(c + dx)} dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] AppellF1(1/2, -1/3, 1/2, 3/2, b*(1-cos(d*x+c))/(a+b), 1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(1/3), x]

[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x]/(d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3))

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rubi steps

$$\int \sqrt[3]{a + b \cos(c + dx)} dx = -\frac{\sin(c + dx) \operatorname{Subst}\left(\int \frac{\sqrt[3]{a + bx}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}}$$

$$= -\frac{\left(\sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)\right) \operatorname{Subst}\left(\int \frac{\sqrt[3]{\frac{a}{-a-b} - \frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\sqrt[3]{\frac{a + b \cos(c + dx)}{-a - b}}}$$

$$= \frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}\sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

Mathematica [A]

time = 0.13, size = 118, normalized size = 1.12

$$\frac{3F_1\left(\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, \frac{a + b \cos(c + dx)}{a - b}, \frac{a + b \cos(c + dx)}{a + b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a + b}} (a + b \cos(c + dx))^{4/3} \csc(c + dx)}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x])^(1/3), x]

[Out] (-3*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(4/3)*Csc[c + d*x])/(4*b*d)

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int (a + b \cos(dx + c))^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cos(d*x+c))^(1/3),x)`

[Out] `int((a+b*cos(d*x+c))^(1/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(1/3), x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")`

[Out] `integral((b*cos(d*x + c) + a)^(1/3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a + b \cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))**(1/3),x)`

[Out] `Integral((a + b*cos(c + d*x))**(1/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c))^(1/3),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(1/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx))^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x))^(1/3), x)

[Out] int((a + b*cos(c + d*x))^(1/3), x)

$$3.59 \quad \int \frac{1}{\sqrt[3]{a + b \cos(c + dx)}} dx$$

Optimal. Leaf size=105

$$\frac{\sqrt{2} F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)} \sqrt[3]{a + b \cos(c + dx)}}$$

[Out] AppellF1(1/2, 1/3, 1/2, 3/2, b*(1-cos(d*x+c))/(a+b), 1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} \sin(c + dx) \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}} F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{d \sqrt{\cos(c + dx) + 1} \sqrt[3]{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(-1/3), x]

[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))

Rule 143

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{a + b \cos(c + dx)}} dx &= -\frac{\sin(c + dx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} \sqrt[3]{a+bx}} dx, x, \cos(c + dx)\right)}{d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\ &= -\frac{\left(\sqrt[3]{-\frac{a + b \cos(c + dx)}{-a - b}} \sin(c + dx)\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} \sqrt[3]{-\frac{a}{-a-b}}}\right)}{d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)} \sqrt[3]{a + b \cos(c + dx)}} \\ &= \frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right) \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)} \sqrt[3]{a + b \cos(c + dx)}} \end{aligned}$$

Mathematica [A]

time = 0.13, size = 118, normalized size = 1.12

$$\frac{3F_1\left(\frac{2}{3}; \frac{1}{2}, \frac{1}{2}, \frac{5}{3}; \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a+b}\right) \sqrt{\frac{b(-1 + \cos(c + dx))}{a+b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a+b}} (a + b \cos(c + dx))^{2/3} \csc(c + dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x])^(-1/3), x]

[Out] (-3*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x])/(2*b*d)

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(dx + c))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(d*x+c))^(1/3),x)`

[Out] `int(1/(a+b*cos(d*x+c))^(1/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(-1/3), x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")`

[Out] `integral((b*cos(d*x + c) + a)^(-1/3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + b \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))**(1/3),x)`

[Out] `Integral((a + b*cos(c + d*x))**(-1/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(-1/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cos(c + dx))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x))^(1/3),x)

[Out] int(1/(a + b*cos(c + d*x))^(1/3), x)

$$3.60 \quad \int \frac{1}{(a+b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=105

$$\frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} \sin(c+dx)}{d \sqrt{1 + \cos(c+dx)} (a+b \cos(c+dx))^{2/3}}$$

[Out] AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2^(1/2)/d/(a+b*cos(d*x+c))^(2/3)/(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} \sin(c+dx) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right)}{d \sqrt{\cos(c+dx) + 1} (a+b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(-2/3),x]

[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b/(b*e - a*f)))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b

*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \cos(c + dx))^{2/3}} dx &= -\frac{\sin(c + dx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} (a+bx)^{2/3}} dx, x, \cos(c + dx)\right)}{d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\ &= -\frac{\left(\left(-\frac{a+b \cos(c+dx)}{-a-b}\right)^{2/3} \sin(c + dx)\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} \left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^{2/3}} dx, x, \cos(c + dx)\right)}{d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)} (a + b \cos(c + dx))} \\ &= \frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)} (a + b \cos(c + dx))^{2/3}} \end{aligned}$$

Mathematica [A]

time = 0.12, size = 116, normalized size = 1.10

$$\frac{3F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a+b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx))}{a+b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a+b}} \sqrt[3]{a + b \cos(c + dx)} \csc(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x])^(-2/3), x]

[Out] (-3*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x])/(b*d)

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(dx + c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(d*x+c))^(2/3),x)`

[Out] `int(1/(a+b*cos(d*x+c))^(2/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(-2/3), x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((b*cos(d*x + c) + a)^(-2/3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))**(2/3),x)`

[Out] `Integral((a + b*cos(c + d*x))**(-2/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(-2/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cos(c + dx))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(1/(a + b*cos(c + d*x))^(2/3), x)
```

3.61 $\int \frac{1}{(a+b \cos(c+dx))^{4/3}} dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{4}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} \sin(c+dx)}{(a+b)d\sqrt{1+\cos(c+dx)} \sqrt[3]{a+b \cos(c+dx)}}$$

[Out] AppellF1(1/2,4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/(a+b)/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} \sin(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{4}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\cos(c+dx)+1} \sqrt[3]{a+b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(-4/3),x]

[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/((a + b)*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))

Rule 143

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

$m, n, p\}, x] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[b/(b *c - a*d), 0] \&\& !\text{GtQ}[b/(b*e - a*f), 0]$

Rule 2744

$\text{Int}[(a + b \sin(c + dx))^n, x_Symbol] \rightarrow \text{Dist}[\text{Cos}[c + dx]/(d \sqrt{1 + \sin[c + dx]} \sqrt{1 - \sin[c + dx]}), \text{Subst}[\text{Int}[(a + b x)^n / (\sqrt{1 + x} \sqrt{1 - x}), x], x, \text{Sin}[c + dx]], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \cos(c + dx))^{4/3}} dx &= -\frac{\sin(c + dx) \text{Subst}\left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} (a+bx)^{4/3}} dx, x, \cos(c + dx)\right)}{d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\ &= -\frac{\left(\sqrt[3]{\frac{a + b \cos(c + dx)}{-a - b}} \sin(c + dx)\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} \left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)} dx, x, \cos(c + dx)\right)}{(a + b) d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)} \sqrt[3]{a + b \cos(c + dx)}} \\ &= \frac{\sqrt{2} F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}} \sin(c + dx)}{(a + b) d \sqrt{1 + \cos(c + dx)} \sqrt[3]{a + b \cos(c + dx)}} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(110) = 220.

time = 1.37, size = 268, normalized size = 2.44

$$\frac{15aF_1\left(\frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{a + b \cos(c + dx)}{a + b}\right) \sqrt{\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{\frac{b(1 + \cos(c + dx))}{a - b}} (a + b \cos(c + dx)) \csc(c + dx) - 6 \left(5b^2 + 2F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{a + b \cos(c + dx)}{a + b}\right) \sqrt{\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a + b}} (a + b \cos(c + dx))^2 \csc^2(c + dx)\right) \sin(c + dx)}{10b(a^2 - b^2)d\sqrt[3]{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x])^(-4/3), x]

[Out] (15*a*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*(a + b*Cos[c + d*x])*Csc[c + d*x] - 6*(5*b^2 + 2*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^2*Csc[c + d*x]^2*Sin[c + d*x])/(10*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(1/3))

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(dx + c))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(d*x+c))^(4/3),x)`

[Out] `int(1/(a+b*cos(d*x+c))^(4/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(4/3),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(-4/3), x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(4/3),x, algorithm="fricas")`

[Out] `integral((b*cos(d*x + c) + a)^(2/3)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))**(4/3),x)`

[Out] `Integral((a + b*cos(c + d*x))**(-4/3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c))^(4/3),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(-4/3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cos(c + dx))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x))^(4/3),x)

[Out] int(1/(a + b*cos(c + d*x))^(4/3), x)

3.62 $\int (a + b \cos(c + dx))^n dx$

Optimal. Leaf size=103

$$\frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right) (a + b \cos(c + dx))^n \left(\frac{a+b \cos(c + dx)}{a+b}\right)^{-n} \sin(c + dx)}{d \sqrt{1 + \cos(c + dx)}}$$

[Out] AppellF1(1/2, -n, 1/2, 3/2, b*(1-cos(d*x+c))/(a+b), 1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^n*sin(d*x+c)*2^(1/2)/d/(((a+b*cos(d*x+c))/(a+b))^n)/(1+cos(d*x+c))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2744, 144, 143}

$$\frac{\sqrt{2} \sin(c + dx)(a + b \cos(c + dx))^n \left(\frac{a+b \cos(c + dx)}{a+b}\right)^{-n} F_1\left(\frac{1}{2}; \frac{1}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right)}{d \sqrt{\cos(c + dx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^n, x]

[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^n*Sin[c + d*x])/(d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^n

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx))^n dx &= -\frac{\sin(c + dx) \operatorname{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\ &= -\frac{\left((a + b \cos(c + dx))^n \left(-\frac{a+b \cos(c+dx)}{-a-b}\right)^{-n} \sin(c + dx)\right) \operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\ &= \frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a+b}\right) (a + b \cos(c + dx))^n \left(\frac{a+b \cos(c + dx)}{a}\right)}{d\sqrt{1 + \cos(c + dx)}} \end{aligned}$$

Mathematica [A]

time = 0.16, size = 121, normalized size = 1.17

$$\frac{F_1\left(1 + n; \frac{1}{2}, \frac{1}{2}; 2 + n; \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a+b}\right) \sqrt{\frac{b(-1 + \cos(c + dx))}{a+b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a+b}} (a + b \cos(c + dx))^{1+n} \csc(c + dx)}{bd(1 + n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^n,x]
```

```
[Out] -((AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(1 + n)*Csc[c + d*x])/(b*d*(1 + n)))
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int (a + b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^n,x)
```


[Out] $\text{int}((a+b*\cos(d*x+c))^n,x)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b*\cos(d*x+c))^n,x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((b*\cos(d*x + c) + a)^n, x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b*\cos(d*x+c))^n,x, \text{algorithm}="fricas")$

[Out] $\text{integral}((b*\cos(d*x + c) + a)^n, x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cos(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b*\cos(d*x+c))^{**n},x)$

[Out] $\text{Integral}((a + b*\cos(c + d*x))^{**n}, x)$

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b*\cos(d*x+c))^n,x, \text{algorithm}="giac")$

[Out] $\text{integrate}((b*\cos(d*x + c) + a)^n, x)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*\cos(c + d*x))^n,x)$

[Out] $\text{int}((a + b*\cos(c + d*x))^n, x)$

Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```

```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```



```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
#do NOT call ExpnType() if leaf size is too large. Recursion problem
if leaf_count_result > 500000 then
      return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result := ExpnType(result);
      ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex

```



```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```

```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

4.2.4 SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```



```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```